A STUDENT'S MANUAL

OF A

LABORATORY COURSE

IN

PHYSICAL MEASUREMENTS

REVISED EDITION

BY

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PREFACE

The first edition of this manual was published in 1893. Since then the course for which it was intended has undergone many changes. The present edition embodies these changes. This course, known as Physics C in Harvard College, is also given in the Summer School of the University. As given in the college it is accompanied by a series of illustrated lectures to the class as a whole, one each week, and one hour a week of lectures, recitation, and problem work to the class in small sections. A knowledge of algebra and plane geometry, and a slight acquaintance with the notation of trigonometry is necessary. It should properly be preceded also by a more elementary course in physics, either by laboratory or text-book, preferably the former.

This manual, intended for students' use, has been given the form of an abstract of the daily lectures preceding the laboratory work and describing the experiments to be performed. It is intentionally condensed. For a number of the experiments blue-print directions in regard to special points are placed beside the apparatus. Experience with the first manual showed the necessity of thus giving flexibility to the course, allowing changes in apparatus and gradual growth. Too specific instruction in the manual seems in time not only to deprive the student of initiative but also to make any departure in the apparatus confusing.

CONTENTS

LABORATORY PHYSICS

INTRODUCTION

An effort is here made to explain all the corrections to be applied, and to call attention to all the precautions which should be taken in the accurate and proper performance of the experiments. On the other hand, in the majority of cases the description is purposely not such as will admit of a mechanical and unintelligent interpretation.

A precaution once noted is not again mentioned, but should always be taken wherever applicable. The number of observations which should be made in each case is not specified, but in regard to this the following may be of interest. One observation has no check against blunders and as a rule contains in itself no criterion of its accuracy. A greater number of observations supplies this deficiency and diminishes the probable error of the mean. Errors of observation, and other errors as liable to be above as below the true value, are alone diminished in this way. Instrumental errors cannot thus be eliminated except by changing in each observation the conditions of the experiment; for example, in determining the zero of the spherometer, by using first one side and then the other of the plate glass, and by using different parts of the surface; or in the experiment with the spectrometer, by using different parts of the graduated circle. By taking a large number of observations, therefore, the student will not only become more familiar with the principles of the experiment and the handling of the apparatus but will also increase the accuracy of his results. For a more detailed discussion of the precision of measurement as depending on the number of observations see the Appendix.

While accuracy in measurement is desirable, precision of statement is no less so, and as a violation of this must be included the disregard of the number of figures which are significant. If three measurements of the length of a rod are 12.67, 12.64, and 12.66 cm., their sum is 37.97. The student, on dividing this number by three to find the average, might write it 12.65666 indefinitely, as if the mere process of dividing by three would enable him to measure to the ten- and hundred-thousandth of a centimeter. Of this number only the first four figures are significant, and to keep more is claiming an accuracy unattained. The same mistake is often made in multiplying, the percentage error of the least accurate factor determining in general the percentage error of the result. For example, it is desired to find the volume of a rod, and rough measurements have been made of its dimensions, giving for the length 498 cm. and for its diameter 2.4 cm. The calculation of the volume of the rod based on these measurements would give

$$V = l\pi r^2 = 498 \times 3.1\left(\frac{2.4}{2}\right)^2 = 2223.072 \text{ cc.}$$

In truth, however, only the first two figures are known and the result should be written 2200 cc. If now greater care is taken in measuring and the diameter of the rod is found to be 2.37 cm., the student is justified in keeping another figure in the value of π and another figure in the result, and the latter may be written 2190 cc. Still further care in measurement might justify keeping still another figure in the result. The record should, on the other hand, show all the accuracy attained. If several succeeding figures are known to be ciphers, they should be entered as such, although decimals. For example, if in weighing with balances sufficiently sensitive to detect milligrams the weight of a body is exactly 12 grams, it should be entered as 12.000 grams.

It is well to note in this connection that, while the final result should have the highest degree of accuracy attainable, it does not follow that every factor should be determined with the greatest

possible degree of accuracy. For example, in the above case it is wholly unnecessary to determine the length of the rod to tenths of a centimeter, although with care it may readily be done. In fact, so limited is the accuracy of the other factor that the result would not be affected by an error of a whole centimeter in the measurement of the length. Whatever time can be saved at this point might profitably be devoted to the determination of the diameter of the rod with more care and a greater number of observations.

Forms and blanks for each experiment, to be filled by the student, are not recommended for the keeping of notes. On the other hand, a system of the student's own devising is desirable and should cover the following points: date, object of experiment, apparatus (name and number), method, data, calculation, and results. Under the head of data should be entered all observations however seemingly unimportant. For instance, in weighing, the separate excursions of the pointer to right and left should always be preserved.

Observations should not be entered at random over the page, but according to a carefully planned system; and each should be accompanied by a memorandum so full that another person in reviewing the notes could, at a glance, determine its meaning, of what it is a measure, and in what units it is expressed. This above all things is essential. It is desirable that the observations and calculations should admit of ready comparison one part with another. The notes should therefore be compact and well arranged, and the writing and figuring closely spaced.

In order to avoid continual repetition in the following pages all measurements will be assumed to be in the centimeter-gram-second (C.G.S.) system, and temperature observations will be on the Centigrade scale.

In higher experimental work, whether research or practical, the investigator is determining a quantity whose exact value is unknown to him. Therefore he cannot ask himself whether his results are correct and in agreement with known values, but

whether his method is correct and his observations well taken. Nevertheless the approximate magnitude of the result is often known. This knowledge and the criterion of common sense applied to the results will often avoid gross blunders.

Whether the observations are used or discarded they should all be preserved, and if discarded in the calculation the reason should be entered in the notes. The only observations which need not be preserved are those which the observer has taken for the sake of practice with the express intention of afterwards discarding them.

PART I. EXPERIMENTS IN MECHANICS

1. MEASUREMENTS OF LENGTH

Vernier Gauge

The vernier gauge, an instrument for the measurement of length, is provided with two jaws between which is placed the object to be measured. One jaw is permanently fixed to a metric scale upon which the other slides. On the movable jaw is a mark which should coincide with the zero of the scale when the jaws are together; and when the object to be measured is placed between them this line marks on the scale their distance apart, and thus indicates the length of the object. After recording the number of whole centimeters and millimeters, if a fraction of a millimeter remains it also may be read off directly and accurately by means of a subsidiary scale called a vernier. This is a small scale graduated upon the movable jaw, beginning with the mark above mentioned. If the vernier is to read to tenths of a millimeter against a millimeter scale, it is nine millimeters long, but is subdivided into ten parts. Each division of the vernier, therefore, is shorter than the division opposite it on the main scale by one tenth of a millimeter. Along the vernier each succeeding line will be nearer its corresponding line on the main scale by one tenth of a millimeter. Thus the number of the vernier line, which exactly coincides with a line on the main scale, will be the value in tenths of a millimeter of the fraction which is being measured. In the diagram the reading is 1.66 cm.

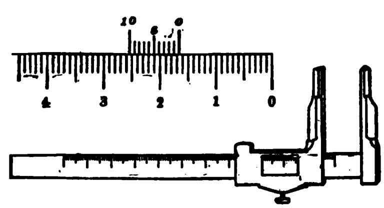

FIG. 1

A more careful examination of the vernier will give some information also in regard to the next decimal place.

The zero error to be applied as a correction to all measurements should be carefully determined by taking the reading when the jaws are pressed gently together. A plus or a minus sign should be prefixed to indicate whether the correction is one to be added or subtracted. Holding the instrument to the light, note at what point the jaws touch, and use this part of the gauge in subsequent measurements.

Measure the length and several diameters of an aluminum cylinder, and calculate its volume. $V(\text{cylinder}) = l\pi r^2$.

In regard to the number of figures which should be retained in the result, see Appendix I.

Micrometer Gauge

The micrometer gauge, although not as convenient as the vernier gauge and incapable of measuring as great lengths, is nevertheless a more accurate instrument. It consists of a screw moving in a nut toward or away from a fixed stop. One complete turn of the screw advances or withdraws its end by an amount equal to the distance between the threads of the screw, — a fraction of a turn by a proportional amount. The number of whole turns may be read on a longitudinal scale, and the fraction of a turn by graduations upon the head of the screw. The object to be measured is placed between the stop and the end of the screw, and the latter turned down upon it. The length of the object is then read off directly in units which depend on the pitch of the screw. Determine the zero error, before and after measuring, by taking the readings with the jaws closed. Apply this, with the proper sign, to all readings. If the instrument is provided with a friction or ratchet head, always take hold of it in turning down the screw, thus securing uniform pressure. If a

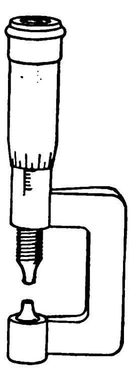

Fig. 2

ratchet head is being used, it should be turned until the ratchet just begins to act, but it should not be allowed to turn further, as the pounding would increase the pressure.

Measure three mutually perpendicular diameters of each of a set of steel bicycle bearing balls, and calculate the total volume of the balls. V (sphere) $= \frac{4}{3}\pi r^3$.

Spherometer: Radius of Curvature

A spherometer, in its essential features, consists of a screw moving in a nut mounted on three legs. It is a form of micrometer in which the stop is replaced by the plane passing through the points of the three legs. The zero reading may be obtained by placing the instrument on a clean piece of plane plate glass and turning down the center screw until its point just touches the surface lightly. Repeat several times on each side of the glass in order to correct for possible warping.

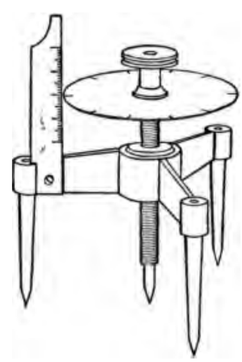

Fig. 3

Place some thin object (a microscope cover glass) upon the plate glass and lower the screw upon it. The difference between this reading and the zero reading is the thickness of the object in units which are determined by the pitch of the screw.

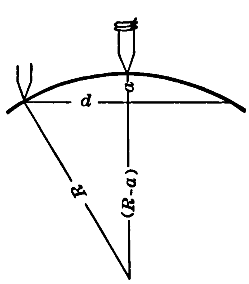

Fig. 4

Place the instrument upon a spherically convex surface and read. If we call a the difference between this and the zero, and d the average distance from the center screw to the three legs, the accompanying diagram will represent the arrangement. By geometry,

$$R^2 = (R - a)^2 + d^2, \quad R^2 = R^2 - 2\,Ra + a^2 + d^2, \quad R = \frac{a^2 + d^2}{2\,a}.$$

d may be obtained by making an impression of the four points upon a piece of paper placed upon a plane hard surface, and measuring the distances with a vernier gauge. Measure separately the radius of curvature of each surface of a double convex lens.

It is obvious that, with a slight modification of the above formula, the instrument may be used for the measurement of concave surfaces.

2. NICHOLSON'S HYDROMETER

Temperature Curve

Nicholson's hydrometer is designed for use as a hydrostatic balance in determining the weight and density of a solid. Preliminary to this use, however, it is necessary to determine the weight which added to the upper scale pan will sink the instrument in water to a given mark on the stem. This depends upon the temperature, for the principle of the instrument is that the weight of a floating body is equal to the weight of the liquid displaced; and on an increase of temperature both the metal of the hydrometer and the water expand, the expansion of the former tending to increase the volume of the displaced water and that of the latter to diminish its density. The two expand unequally. As a result the weight of the displaced liquid varies, and as the weight of the instrument must be equal to this, the weight added to the upper scale pan must be varied accordingly.

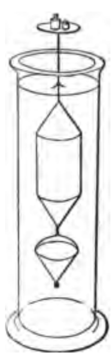

Fig. 5

Find the weight which must be added to the upper scale pan in order to sink the hydrometer to a given mark on the stem in water at about the temperatures 5°, 10°, 15°, 20°, 25°, and 30° C., noting, of course, not the approximate temperature sought but the exact temperatures attained. Fractions of centigrams may be estimated by interpolation. Plot the results

on coördinate paper, taking temperatures for abscissæ and weights for ordinates (see Appendix II).

Care must be taken that the hydrometer is freed from all adhering air bubbles, that it does not touch the sides of the vessel, and that the upper scale pan is dry. The last two conditions may be secured by guiding the stem through a torn paper placed around it and resting upon the top of the jar. The stem of the instrument should be freed from oil by careful cleaning with a weak solution of caustic potash (or of soap), thereby reducing to a minimum the irregularity of the capillary action of the water on the stem. The water should be kept thoroughly stirred that it may be of uniform temperature throughout. In judging the level of the water on the stem look through the water, sighting along the under surface.

Specific Gravity of a Solid by Nicholson's Hydrometer

The specific gravity of a solid is the ratio of its weight to the weight of an equal volume of water. It may be determined by means of the hydrometer, by weighing the solid first in the upper scale pan, and then when submerged in water and resting on the lower pan. When submerged it weighs less, being lifted in part by the water. The loss of weight when in water is equal to the weight of an equal volume of water.

Determine by direct experiment, or by reference to the curve obtained above, the weight which added to the upper scale pan of the hydrometer sinks it to the mark on the stem. Place the object to be weighed, a brass disk, in the upper scale pan and add weights until the hydrometer sinks to the mark on the stem. The total weight in the upper scale pan has just been determined from the curve. By subtracting from this the part which is known, determine the weight of the brass disk. Place the disk on the lower scale pan, again add weights to the upper scale pan, and, repeating the above process, determine the weight in

water of the brass disk. Subtract from the weight of the disk its weight when submerged in water; the difference is the weight of an equal volume of water. Divide the weight of the brass disk by the weight of the water and find the specific gravity of the brass.

In the centimeter-gram system this result will also be equal numerically to the number of grams of brass occupying one cubic centimeter of space.

Strictly, the above experiment should have been performed in water at a temperature of 4° C., but it is so difficult to secure this temperature and to maintain it constant, that it is far better to have the water at nearly the temperature of the room. The error introduced in this experiment is of barely perceptible magnitude.

3. THE EQUAL-ARM BALANCE

Reading by Vibrations

Since time is consumed in waiting for the pointer of an ordinary arm balance to cease swinging, it is best to determine the

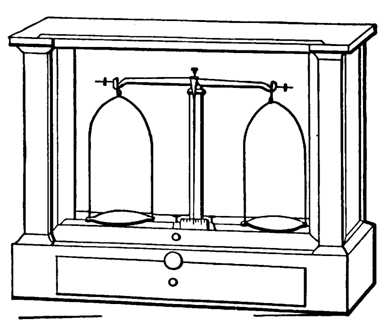

Fig. 6

point at which it would come to rest by observing its turning points. The mean between the average of the readings to the right and the average of those to the left, of several consecutive

swings, will be the point of rest. To correct for the error due to the diminution of the amplitude of swing by friction, take an uneven number of swings, thus throwing both the greatest and the least upon the same side. If the pointer moves over a small scale numbered from 0 to 20, suppose the consecutive excursions are as follows.

To Left	To Right
8.6	
	12.6
8.9	
	12.2
9.3	
3)26.8	2)24.8
8.9	12.4
	8.9
	2)21.3
	10.6, point of rest

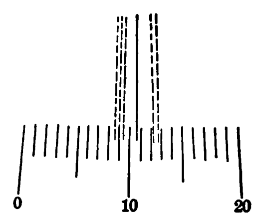

Fig. 7.

Determine the zero point (i.e. with no load in either scale pan) by means, separately, of three, five, and seven swings with an amplitude (from the one side to the other) of about three divisions. Then once (five swings) with an amplitude of six or seven divisions, and again with one of nine or ten. Finally, allow the pointer to come to rest and note its final position.

The balance should be so leveled upon the table that it cannot rock and thus change the zero point during the progress of the experiment. Disturbing air currents, which will make the balance swing irregularly, may be avoided by always lowering the front of the case while weighing. As the pointer is some distance in front of the scale its reading will depend somewhat upon the position of the observer. Error from this source may be avoided by placing a small mirror behind the pointer, and always taking such a position that the latter hides its own image.

The zero point is to be redetermined each day before beginning work with the balance; usually two sets of observations are sufficient.

Double Weighing

In accurate work with balances it is necessary to correct for inequality in the lengths of the arms. The ratio of the lengths may be determined and the correction applied by the following method. Let the lengths of the arms be a (left) and b (right).

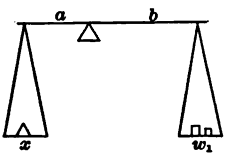

Fig. 8

Place the body to be weighed, an aluminum cylinder, for example, in the left scale pan. By trial determine what weight in the opposite scale pan will bring the pointer nearest to the true zero point previously obtained, and determine the point of rest by vibrations. If this is not exactly the zero, balance is not quite obtained. Add one centigram. This will perhaps send the pointer to the other side of the zero. One weight is too small and the other is too large, but the exact weight required can be found by the following process of interpolation. The difference between the two readings (the distance that the pointer is moved by one centigram) is the sensitiveness of the balance; its reciprocal is the fraction of a centigram required to move the pointer one division; the latter multiplied by the distance from the first point of rest to the zero point is the fraction of a centigram which should have been added to secure exact balance; for example, if the zero point is 10.5, and if the point of rest is 11.2 when the weight is 55.16 g., and 9.8 when the weight is 55.17 g., one centigram moves the pointer 1.4 divisions, the weight necessary to move it 1 division being $\frac{1}{1.4}$ of a centigram; and the weight necessary to move it from 11.2 to 10.5, that is, .7 of a division, is $\frac{.7}{1.4} = .5$ of a centigram, giving for the true weight necessary to secure balance $w_1 = 55.165$ g. This method, called interpolation, makes it possible to weigh to milligrams, although the smallest weight in the set is a centigram.

From the principle of the lever we have

$$ax = w_1 b.$$

Change x to the opposite scale pan and determine the weight w_2 necessary to exactly balance it. The weights now are on the arm a and the equation becomes

$$bx = w_2a.$$

Multiply one equation by the other.

$$abx^2 = w_1w_2ab;$$

whence $$x = \sqrt{w_1w_2},$$

or $$x = \frac{w_1 + w_2}{2},$$

approximately, since w_1 and w_2 differ but slightly. Calculate x.

Divide one equation by the other and we have

$$\frac{a}{b} = \frac{w_1b}{w_2a},$$

or $$\frac{a}{b} = \sqrt{\frac{w_1}{w_2}} = 1 + \frac{w_1 - w_2}{2\,w_2}, \text{ approximately.}$$

From this calculate the ratio of the lengths of the arms.

This process of weighing a body, first in one scale pan and then in the other, to eliminate the error arising from the inequality in the lengths of the arms of the balance, is called double weighing.

Because it may be of value in future work note the temperature of the room and the barometric pressure during this experiment.

Weight in Vacuo and Density of a Solid by Submersion

The operation of weighing consists, strictly, in finding the number of standard masses called grams which are attracted by the earth with as much force as is the body which is being weighed. If a weighing has been made in air by means of the equal-arm balance, it does not follow that the earth is attracting with equal force the weights and the body which is being weighed, even after a correction has been applied by double weighing for any accidental inequality in the lengths of the arms. The experiment is complicated by the buoyant action of the air which tends to lift not only the body which is being weighed but also the brass

weights. Properly, therefore, the experiment should be performed in vacuo. It is, however, usually more convenient to work in the air and then apply the necessary corrections. The buoyant force of the air, the lifting force which it exerts, is equal to the weight of the air displaced.

It is desired to find from the result of any weighing the number of grams which would have balanced the cylinder had the experiment been performed in vacuo. For a moment regard only the buoyant force exerted by the air on the brass weights. Had the experiment been performed in vacuo not so many weights would have been required, since they would not be lifted in part by the air. The first correction, therefore, is to *subtract* from the above result the weight of the air displaced by the brass. To find the value of this correction calculate first the volume of the brass by dividing the number of grams by the number of grams required to occupy one cubic centimeter of space. Multiply this, the volume of the air displaced, by the weight of each cubic centimeter of air as determined from table 1 given in Appendix III. The result is the correction to be subtracted. On the other hand, the cylinder weighed has also been buoyed up by the air, and consequently appears lighter than when weighed in vacuo. A second correction must therefore be applied by *adding* the weight of the air displaced by the cylinder.

The result thus corrected is the number of grams which are attracted by the earth with the same force as is the aluminum cylinder. This is the knowledge that is almost always sought in weighing; and strictly no accurate weighing is complete until these corrections have been applied. If the object being weighed is of irregular shape, a special method, as explained below, must be employed to find its volume.

The density of any substance is its mass (sometimes loosely called matter) per unit volume. Thus the mean density of any body is found by dividing its mass by its volume. $D = \frac{M}{V}$.

If the volume is not known, and the body is often of such irregular shape that it cannot be determined by direct measurement,

the following process gives first the volume approximately and then finally the volume far more accurately than it can be measured by the vernier gauge, even under the most favorable conditions as to shape. This process then furnishes the denominator of the density fraction.

The aluminum cylinder having been weighed by vibrations, interpolation, and double weighing, a correction for the buoyancy of the air on the weights is to be immediately applied to the result of the weighing as explained above. This correction is not only to be applied to all weighings throughout this experiment but to all future weighings performed in air and justifying this degree of care. Tie around the cylinder a harness of fine iron wire and suspend it from the top of the stirrup of the balance in a beaker of water placed on a bridge over one of the scale pans. Weigh to milligrams by interpolation and record the temperature of the water. While being weighed the cylinder should be wholly submerged by the water and only one strand of wire should pass through the surface. Remove the cylinder and weigh the wire harness, adding water until the wire is submerged to the same level. The difference between the last two weighings will be the weight of the cylinder in water. Subtract the weight of the cylinder in water from its weight in air and the result will be approximately the weight of the water displaced, and in the centimeter-gram system this will be an *approximate* volume. With this find the weight of the air displaced by the cylinder, as in the last experiment, and add it to the weight of the cylinder in air. This will give the weight of the cylinder in vacuo, numerically equal to its mass, and the numerator of the density fraction. Now subtract from the weight in vacuo the weight in water and the result will be the exact weight of the water displaced. Multiply this by the volume of each gram of water (Appendix III, table 2). The result will be the exact volume of the water displaced, and therefore the volume of the cylinder and the denominator of the density fraction.

Calculate from this the mean density of the aluminum.

4. DENSITY OF A SOLID

Weigh a number of pieces of aluminum wire sufficient in amount to nearly fill a specific-gravity bottle. Weigh this bottle filled with water, noting the temperature of the water. Place the solid just weighed in the bottle; fill with water and reweigh. From these three weighings, making all necessary corrections for the buoyancy of the air on the weights and on the aluminum, and for the temperature of the water, determine the density of the aluminum, having due regard to significant figures in the result.

5. SIMPLE PENDULUM: FORCE OF GRAVITY

The absolute unit of force (called a dyne) is such that, acting for one second on a mass of one gram, unconstrained, will give it a velocity of one centimeter per second. The earth's attraction (gravity) acting alone for one second on a gram near the earth would give it a comparatively great velocity. If the velocity of a body falling freely were measured at the end of the first second, it would be a direct determination of the force of gravity in absolute units. It is difficult to do this accurately. A more ready, though indirect, method of measuring the force of gravity is by the simple pendulum.

The ideal simple-gravity pendulum is one in which the swinging body is concentrated at a point and is suspended by a weightless thread. The time of a single swing of such a pendulum depends upon the length of the pendulum and the force of gravity. The relation is expressed by the formula

$$t = \pi \sqrt{\frac{l}{g}}, \text{ or, transposing, } g = \pi^2 \frac{l}{t^2},$$

where t is the time in seconds of a single swing from one side to the other, or from the center out to one side and back to the

center again, where l is the length of the pendulum and where g is the force of gravity on a one-gram mass measured in absolute units.

In this experiment for the determination of g the pendulum may be of any length. It is, however, possible to time the pendulum with greater accuracy and ease when its periodic time is nearly that of a break-circuit clock, namely a second. The length of the pendulum will therefore be taken at about one meter, and the method of timing employed will be that known as the method of coincidences, which may be arranged as shown in the adjacent diagram. The break-circuit clock, through a primary relay, sends a momentary current each second through an electromagnetic device which opens a slit and produces an exceedingly short flash of light. This slit is placed immediately behind the pendulum and carefully adjusted so that the ball completely and centrally covers it when hanging at rest. The pendulum is then started. As the pendulum swings it will, part of the time, be in front of, part of the time to one side of, the slit when the flash occurs. When the pendulum is in front of the slit as the flash occurs the latter will be eclipsed. From one period of eclipse to the next the pendulum will have lost or gained one vibration on the clock pendulum — whether lost or gained can be easily seen by inspection. As the pendulum continues to swing, note the time of day — hours, minutes, and seconds — of each eclipse, or coincidence, through a considerable lapse of time. If t seconds have elapsed during n intervals between eclipses, then the pendulum will have made $t \pm n$ swings, and the time of a single swing will be $t \div t \pm n$.

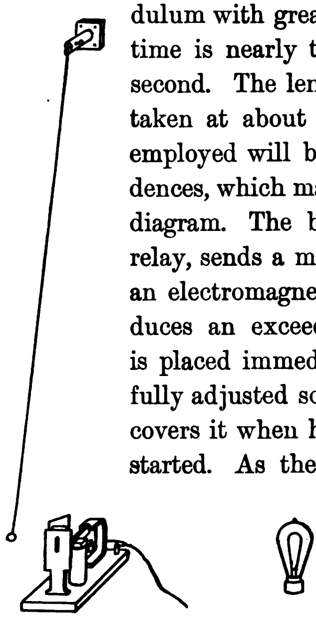

Fig. 9

Suspend a small lead ball by a fine silk thread 100 cm. long, clamping the upper end of the thread firmly. Let the length of

the pendulum be accurately 100 cm. from the lower edge of the clamp to the upper surface of the ball. The pendulum should be allowed to hang until all the stretch has been removed from the thread and then again adjusted. The length of the pendulum should always be measured at the beginning and at the end of the experiment. The total length of the pendulum, the l to be used in the above formula, is from the point of support to the center of the ball. It is therefore the length of the thread plus the radius of the ball determined by a vernier gauge.

Determining in this way the length and time, the latter determined when the pendulum is swinging over an arc of about 10°, calculate the value of g. This constant should be determined by repeated trials and with great accuracy, as it enters into the calculation of subsequent experiments.

The above formula $t = \pi \sqrt{\frac{l}{g}}$ may be proved in the following way.

Disregarding friction, a body falling from one level to another has at the second level the same kinetic energy regardless of the path it may have followed, for it has in every case lost the same amount of potential energy. We can therefore calculate the velocity of the pendulum bob at any point of its arc by calculating the velocity which it would have if it had fallen vertically to that level from the initial level. For this purpose we have the formula for a falling body, $v = \sqrt{2\,gs}$, when s is the force of gravity in dynes and s the vertical distance. Referring to the adjacent figure for the significance of the letters, the velocity V of the pendulum at the middle of its swing is

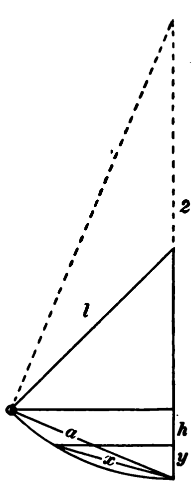

Fig. 10

$$V = \sqrt{2\,gh},$$

where, by similar triangles,

$$h : a = a : 2\,l,$$

or

$$h = \frac{a^2}{2\,l};$$

whence
$$V = \sqrt{\frac{a^2 g}{l}}.$$

Similar reasoning gives for the velocity v at any point on the arc, at a distance say x from the middle,

$$v = \sqrt{\frac{(a^2 - x^2)g}{l}}.$$

This gives the velocity of the pendulum at any point of its arc. It is interesting to show that this motion is the projection upon a diameter of uniform circular motion. Imagine a wheel of radius a revolving in a horizontal plane with a constant circumferential velocity of $\sqrt{\frac{a^2 g}{l}}$. The component of this velocity parallel to the motion of the pendulum is obviously, by similar triangles,

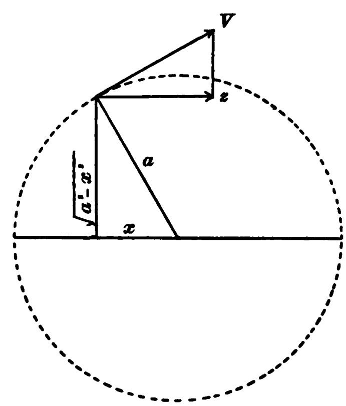

FIG. 11

$$z : \sqrt{\frac{a^2 g}{l}} = \sqrt{a^2 - x^2} : a,$$

or
$$z = \sqrt{\frac{g(a^2 - x^2)}{l}}.$$

z is therefore equal to v, the velocity of the pendulum at the corresponding point. The point on the circumference will therefore move around the semicircle while the pendulum moves from side to side. The time of a single swing of the pendulum is therefore

$$t = \frac{\pi a}{\sqrt{\frac{a^2 g}{l}}}, \quad \text{or} \quad t = \pi \sqrt{\frac{l}{g}}.$$

The formula can be proved much more simply and directly by means of calculus.

6. TORSION PENDULUM: MOMENT OF TORSION AND MOMENT OF INERTIA

In producing rotary motion one is concerned with the moment of the force and the moment of inertia; in the torsion pendulum, for example, with the moment of torsion of the wire and the moment of inertia of the suspended body.

The power of a force to produce rotary motion depends not only on the magnitude of the force but also on the perpendicular distance from its point of application to the axis about which the rotation occurs. This rotating power is called the moment of the force, and is equal numerically to the product of the force and the distance. In discussing rotary motion it is convenient to adopt as a unit angle one in which the length of the arc is equal to the radius. As may be readily seen, this unit angle is equal to a little over 57°. If one end of a straight wire be twisted, the other end being rigidly clamped, the wire will exert a force tending to untwist itself. The moment of this force when the end of the wire is turned through the unit angle is called the moment of torsion of the wire.

The inertia of a body is that universal property of matter by virtue of which it obeys Newton's law, that at rest it will remain at rest, in motion it will continue in motion until acted upon by some external force. The inertia of a body is numerically equal to the force required to give it the unit linear velocity in one second, and this is in turn equal to its mass. Inertia and mass are thus numerically equal. In passing to the consideration of rotary motions not only the force but also the inertia must be regarded as having a certain moment about the axis of rotation. The moment of inertia of a body to rotary motion is numerically equal to the moment of the force required to give it in one second the unit angular velocity. The following discussion will show how this may be calculated. From the definition of the unit angle given in the last experiment it follows that the unit angu-

lar velocity is such that every point in the body moves with a linear velocity equal to its distance from the axis of rotation. To start with the simplest case, conceive the body to consist of a mass m concentrated at a point and made to rotate at the end of a weightless arm of length r. When it has the unit angular velocity it will have a linear velocity of r centimeters per second. By definition the force in dynes which must be applied for one second directly to the body to give it this velocity is equal to mr. The moment of this force is $mr \times r$, or mr^2, and this is equal numerically to the moment of inertia of the body. When the body is so large that it cannot be considered as concentrated at a point, it must be conceived to be divided into a great number of very small elements. The moment of inertia of the body is equal to the sum of the moments of inertia of the separate parts, $I = m_1r_1^2 + m_2r_2^2 + m_3r_3^2$, etc. It follows from this, that the moment of inertia of a radially thin ring, rotating in its own plane about its center, is equal to its mass multiplied by the square of its radius, since r is the same for every element; and that if I_1 and I_2 are the separate moments of inertia of two bodies, their combined moment of inertia is $I_1 + I_2$.

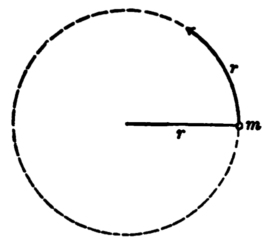

FIG. 12

If a body be suspended by a wire, twisted, and then released, it will oscillate with a rotary motion. This constitutes a torsion pendulum, and the formula expressing its various relations is

$$t_1 = \pi \sqrt{\frac{I_1}{T}}.$$

t_1 is the time of a single oscillation; T is the moment of torsion of the suspending wire, depending on its length, diameter, and material; and I_1 is the moment of inertia of the suspended body, depending on its mass, shape, and axis of rotation. If the moment of inertia is changed, the time of oscillation will also change. For example, if to a body of irregular shape and therefore unknown

moment of inertia, I_1, a ring is so added that in oscillating it will rotate about its center, the new moment of inertia will be the sum of the two, $I_1 + I_2$, and the new time of oscillation will be

$$t_2 = \pi \sqrt{\frac{I_1 + I_2}{T}}.$$

From these two equations, I_2 being calculated after weighing and measuring the ring, and t_1 and t_2 being determined experimentally, I_1 and T may be determined.

The object of the present experiment is the determination of the moment of inertia of an irregular suspended body and the moment of torsion of the suspending wire by the method suggested above, and a comparison of the moment of torsion thus determined with the moment of torsion determined by a less accurate but more direct method, — the application of a measured couple to the wire.

The irregular body whose moment of inertia is to be determined is here, as is usually the case in such investigations as the magnetometric determination of the horizontal component of the earth's magnetic force, a support carrying a mirror. This support has a clamp above for fastening it to the wire, while it is expanded below into a disk to carry the ring whose moment of inertia is calculable. The oscillating of the pendulum is observed by means of a telescope and a lamp so placed that a flash of light reflected from a mirror on the pendulum occurs as the pendulum passes through its central position. The timing is done by means of a chronograph and break-circuit clock whose method of use will be obvious on inspection.

Weigh the ring and measure its inner and outer diameters. From its mass and the mean of the squares of its inner and outer radii calculate its moment of inertia. Time accurately with and without the ring in place, and calculate the moment of inertia of the mirror, clamp, support, etc., and the moment of torsion of the suspending wire. The moment of torsion of a wire being

inversely proportional to the length of the wire, calculate the moment of torsion per meter of length.

A very interesting, but not so accurate, direct measurement of the moment of inertia can be made by steadying the bottom of the torsion pendulum by means of a needle working in a jewel bearing, and wrapping around the needle a silk fiber to which a force is applied either by means of a spiral spring or by weights sufficient to turn the pendulum through the unit angle. The combined force of the two ends of the fiber, sum or difference, according as the arrangement of the fiber is such that the two ends pull to produce rotation in the same or in opposite directions, multiplied by the radius of the needle, will give the moment of inertia. In order to compare this with the result obtained above, it is, of course, necessary to express the force in dynes.

The formula for the torsion pendulum can be deduced from the formula for the simple-gravity pendulum, or can be proved directly by a process similar in every step to the proof for the gravity pendulum. It is, of course, most simply proved with the aid of calculus.

PART II. EXPERIMENTS IN SOUND

7. PITCH BY THE MONOCHORD

Both from a physical and a musical standpoint the pitch of a note is its most important characteristic, and depends upon the number of vibrations per second. If the sounding body is a stretched wire (or string), the number of vibrations per second, when the wire is plucked transversely, depends upon its mass, its length, and the tension with which it is stretched. The material of which it is composed does not affect its pitch, except in so far as its density determines the mass. If l denotes the length in centimeters of the vibrating part of the wire, m the mass in grams of each centimeter, and t the tension expressed in dynes, then n, the number of complete vibrations per second, is given by the formula

$$n = \frac{1}{2l}\sqrt{\frac{t}{m}}.$$

Since l, t, and m can be readily determined, the pitch of the note can be calculated. The instrument for this test, usually called a monochord, consists of a resonating chamber, over which a wire is stretched by a known weight. The length of the vibrating part of the wire may be varied at pleasure by means of the movable fret, until the note emitted is in unison with the note to be tested. The pitch of the note can then be calculated.

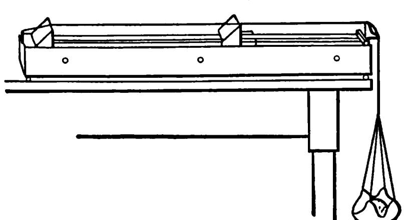

FIG. 13

Weigh a steel wire about a meter and a half long, measure its length, and calculate the mass m per centimeter. Stretch it over

the monochord by a load of about six kilograms, and tune to unison with a tuning fork, varying the length by means of the movable fret. When in unison measure the length l of the vibrating part of the wire. Calculate in dynes the tension with which the wire is stretched, by multiplying the load (in grams) by the value of g found in Experiment 5, including as part of the load the weight of the pad holding the weights. With these values of l, m, and t calculate the pitch number n of the note emitted by the wire by means of the above formula. This will also be the pitch of the fork, since the two are in unison.

Also determine the pitch of the fork, using a different stretching load on the wire.

If the wire on the monochord is stretched over a right-angle lever, in order that the tension in the vibrating part of the wire may equal the load, it is necessary that the vertical and horizontal arms should be of equal length. They are, probably, nearly but not exactly of equal length. The error arising from slight inequality may be eliminated as follows. Tune carefully to unison with the fork and measure the length of the wire. Lift the weights, remove the lever, and turn it over so that the arm which was before horizontal is now vertical. Again tune to unison and measure the length of the wire. This should be repeated several times and the mean of all the lengths taken as the correct value of l to be used in the calculation of n. When the longer arm of the lever is horizontal the tension in the vibrating part is greater than the load; when however the longer arm is vertical the tension is less. Care must be taken throughout the experiment that the lever does not rock so far forward or backward as to rest against one of the shoulders of the recess in which it plays.

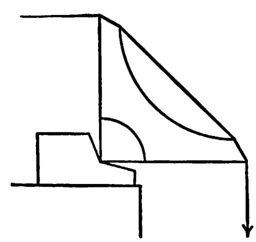

FIG. 14

In many forms of the monochord the wire is stretched over a second fret and then over a pulley. In this case the tension in the vibrating part of the wire is modified by the friction of

the pulley and fret. It is much more difficult to entirely eliminate this error, but it may be greatly reduced as follows. Lift the weights and allow them to settle. The friction will in this case diminish the tension. Tune without disturbing the weights and record the length of wire giving unison with the fork. Now draw down on the weights gently, and, letting go, allow them to rise. In this case the friction increases the tension. Retune by varying the length. This should be repeated several times and the average taken of all the lengths thus determined.

When two notes nearly but not quite in unison are sounding together they at one moment reënforce and then oppose each other; the sound is alternately loud and faint. These pulsations of the sound are technically called *beats,* and furnish a mechanical method of ultimately securing unison. The beats cannot be detected until the wire and the fork are nearly in unison, but when once they are obtained the length of the wire should be varied in such a direction as to make the beats succeed each other more and more slowly, until ultimately indistinguishable. If the fork is not mounted on a resonant box, its sound may be greatly reënforced by holding its shank on the monochord. In this case the beats may be felt by resting the fingers lightly on the box.

The fork should be set in vibration by bowing, or by striking against a piece of firm leather. If the fork is struck against a hard surface, it is liable to become worn and its pitch changed in consequence.

Musical Interval: Scale

The musical scale consists of a series of eight notes whose pitch numbers bear to each other comparatively simple ratios. These ratios determine what are called the musical intervals of the notes. When the musical interval of two notes is simple, that is, when the ratio of the pitch numbers is a simple ratio, as 2 : 1 or 4 : 3, etc., the notes when sounded together give a pleasant sensation and are said to be in harmony or in accord.

The eight notes constituting the scale are denoted, beginning with the lowest, by the letters *C, D, E, F, G, A, B, c*. The musical interval of the eighth note to the first, *c* to *C*, is called an octave; the interval of the fifth to the first, *G* to *C*, is called the interval of the fifth; of *E* to *C*, the interval of the third; and so on.

Stretch the steel wire, preserved from the last experiment, over the monochord by a load of about six kilograms. Taking all precautions, determine by means of this the pitch numbers of each of a set of forks tuned and lettered on the above scale. Having determined the pitch number of each, calculate the ratio expressing the musical interval between each note and the lowest note in the scale, *C*. By an examination of these ratios determine to what simple ratios of whole numbers the interval of the octave, the fifth and the third, may be reduced. In making the reduction consider one vibration in the pitch number as a possible experimental error. Sound in turn the forks *c*, *G*, and *E* with the lower *C* and compare the sensation in each case with the sensation in listening to *B* and *C* sounding together. Reduce the ratio expressing the interval of the latter combination to as simple a form as possible.

Using the results of the above experiment, calculate also the ratio expressing the musical interval between the successive notes *D* to *C*, *E* to *D*, *F* to *E*, etc. That they may be the more readily compared, reduce all the resulting fractions to the same denominator, for example, to 1; in other words express the ratios decimally. Between what notes are the intervals large? When these larger intervals are filled by intermediate notes called sharps and flats the result is called a chromatic scale. In this scale the intervals between successive notes are nearly but not quite uniform.

8. VELOCITY OF PROPAGATION OF SOUND

Velocity of Sound in Air by the Resonance Tube

Sound is transmitted as longitudinal vibrations, with a definite velocity depending upon the elasticity and density of the medium. This velocity may be measured indirectly in the following manner. Hold the fork, whose pitch number n was determined in Experiment 7, immediately over the open end of a large glass tube. The latter should be so arranged that the length of its air column may be varied by a sliding, but tightly fitting diaphragm.

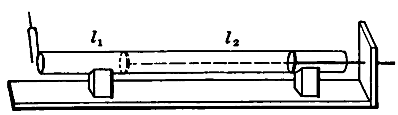

Fig. 15

Starting with the air column short, vary its length until it strongly reënforces the sound of the fork. Mark the point of greatest resonance by a narrow rubber band clasping the tube. The impulse sent down the tube by the forward vibration of the fork travels the length l_1 of the tube, and is reflected back in time to reënforce the fork in its backward motion. Thus during the forward motion of the fork, that is, during half a vibration, the sound travels twice the length of the tube, $2\,l_1$. The distance actually traveled by the impulse in going each way is a little greater than the length of the tube on account of the reflection from the sides and the spreading at the open end. The correction to be added is equal to nearly one quarter the diameter d of the tube. During half a vibration, therefore, the sound goes $2\left(l_1 + \frac{d}{4}\right)$ centimeters, and hence during n whole vibrations, or one second, the sound goes $4\,n\left(l_1 + \frac{d}{4}\right)$ centimeters; whence the velocity of sound in air at the temperature of the room is $V_t = 4\,n\left(l_1 + \frac{d}{4}\right)\cdot$ Continuing to increase the length of the tube, find a new position which will also strongly reënforce the fork. By similar reasoning it may be shown

that the impulse now travels the length of the tube and back in time to reënforce not the next but the next but one backward motion of the fork. The sound therefore travels from the first position of the diaphragm to the second and back again to the first position, $2\,l_2$, during one vibration. During n vibrations, or one second, it would travel n times as far. Measure the distance l_2 between the two positions of the diaphragm.

$$V_t = 2\,nl_2.$$

Note the temperature. From these results calculate the velocity of sound V_0 in air at 0° C.

$$V_0 = \frac{V_t}{\sqrt{1 + .00366\,t}}.$$

For other intermediate positions of the diaphragm slight resonance of the overtones of the fork may be obtained, but they can be distinguished by their higher note. The velocity of sound in air is somewhat affected by the moisture present. If provided with a hygrometer, it is well to record the humidity of the air. The change in velocity of sound due to this will probably not amount to half of one per cent of the whole velocity.

While the air in the longer resonance tube is vibrating with the fork the waves reflected from the closed end of the tube interfere with the succeeding waves coming down the tube at the first position of the diaphragm, causing the air here to remain almost at rest. This point is called a node. The air in the rest of the tube is in vibration to and fro, at a maximum midway between the two positions of the diaphragm and at the open end.

Repeat the experiment, using a fork of a different but known pitch, one from the set used in the last experiment.

Velocity of Sound in Brass

A brass rod, held firmly by the middle and stroked with a resined cloth, emits a clear high note by longitudinal vibration. Each half vibrates as does the column of air in the shorter resonance tube

of the preceding experiment. Sound, therefore, travels in brass sufficient to carry the impulse from the end of the rod to the center and back, a distance equal to the whole length of the rod, during half a vibration.

Hold a quarter-inch brass rod, about 60 cm. in length, in a horizontal position by a vise fixed on the table. Affix to one end of the rod a disk of cork, small enough to slip freely into a large glass tube. Distribute uniformly within the tube, which should be very dry, a small amount of lycopodium powder. Support the tube on V's on the table, and slip it over the rod. The other end of the tube should be closed by the movable diaphragm. Slowly vary the length of the air column in the tube until it is in resonance with the note emitted by the rod when its free end is stroked. This will be shown by the powder quickly drifting into little heaps or ridges. The ridges mark the points along the tube at which the air is stationary, caused by the interference of the waves reflected from the end of the tube with the waves following down the tube. The distance between these points corresponds to the distance between the two positions of the diaphragm in the preceding experiment, and is therefore the distance that the sound would travel in air during half a vibration. But the sound travels in brass the length of the rod during half a vibration. The velocity of sound in brass, therefore, is to its velocity in air, as the length of the brass rod is to the average distance between the nodes or ridges of the lycopodium powder. Calculate the velocity of sound in air, at the temperature of the room, from its velocity in air at 0° C. by the formula $V_t = V_0 \sqrt{1 + .00366\,t}$. Using this and the data above obtained, calculate by the proportion just stated the velocity of sound in brass.

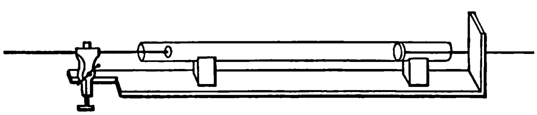

FIG. 16

The clamp may be either a split metal clamp or may be made of a narrow piece of rubber tubing slipped over the rod.

The velocity of sound in any dry gas may be determined by a slight variation of the above experiment. Close the open end of the tube with a cork through which the brass rod can slip loosely, and fill the tube with the gas which is to be experimented upon. Allowing a small stream of gas to flow through the tube during the experiment, adjust the position of the diaphragm until the lycopodium powder collects at the nodes, when the rod is caused to vibrate. The distance between the nodes formed in the gas is the distance that the sound travels during half a vibration of the rod. Comparing this with the distance that the sound has traveled in air in the same length of time in the above experiment, one may readily calculate the velocity of sound in gas, the velocity in air being known.

9. QUALITY BY THE MANOMETRIC FLAME

It is difficult to make a quantitative analysis of musical quality, but the following experiment will show in a qualitative manner the variation in character of tones which are pronouncedly different, such, for example, as the vowels spoken or sung at the same pitch.

In the adjacent diagram *M* represents a revolving mirror in which is viewed a flame represented at *F*. This flame, which escapes from a very small orifice, is fed through a shallow chamber *C*, one side of which is made of thin gold beater's skin. On the other side of the gold beater's skin is a similar shallow chamber *C'* connected by a rubber tube to a mouthpiece, or resonator, into which the vowel or sound to be analyzed is sung. The vibration of the air carried down the tube causes the diaphragm to vibrate, which in turn causes the gas feeding the flame to fluctuate. The height of the flame follows these fluctuations fairly accurately, reproducing

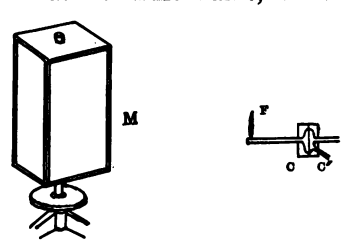

FIG. 17

in its height the original vibration of the air. The flame vibrates so rapidly that its variations cannot be followed directly by the eye, and there is no very apparent change in its character. These fluctuations or vibrations of the flame can, however, be seen when the flame is viewed in the revolving mirror. If the flame were perfectly steady, its virtual image in the mirror would be drawn out into a uniform band by the revolution of the mirror; but when the flame fluctuates the band is serrated, showing in these serrations the character of the vibrations. For best results the speed of the mirror should be adjusted to the pitch of the note. This method of analysis was devised by König.

Using for a mouthpiece a glass tube held near the mouth, observe the character of the vibration for various vowels sounded softly. Make as accurate a sketch as possible of the character of the vibrations, indicating for each the particular vowel sound. Since the character of the vowel sound depends upon the word in which it occurs, make a memorandum of a word which contains the letter as sounded.

Replacing the glass tube by spherical resonators tuned to various pitches, the pitch of each being marked on it, determine the component notes of a few of the vowel sounds by finding the resonators which respond. The vowel must be continually sung (softly) at the same pitch, and that should be the pitch of the lowest resonator in the set, the resonator of which the other resonators are harmonics.

The experiment is at best qualitative and difficult. In order that it should be at all successful, it must be handled with a good deal of care and patience. One precaution that should be noted is that in working with the resonators the mouth should not be held too near, nor should the sounds be too loud, for the result would be to force the vibration in the resonator whether in tune or not. In fact, either with the resonators or with the glass mouthpiece, the experiment is much more successful if the sounds are not loud.

PART III. EXPERIMENTS IN HEAT

10. DETERMINATION OF THE MECHANICAL EQUIVALENT OF HEAT[1]

Description of the apparatus. In the machine for this experiment a vertical spindle carries at its upper end a brass cup *A*. Into an ebonite ring concentric with *A* there fits tightly one of a pair of hollow truncated cones. The second cone fits into the

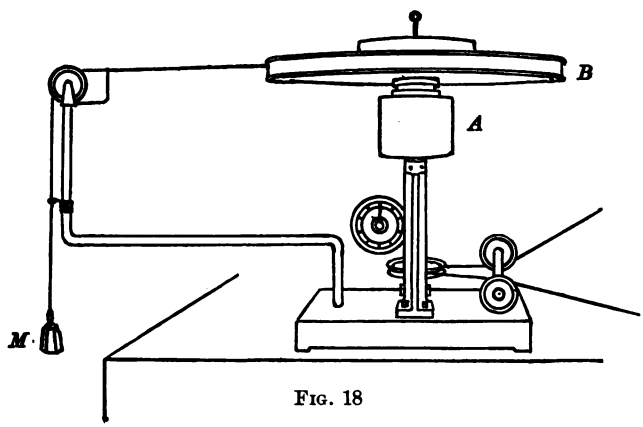

FIG. 18

first one and is provided with a pair of steel pins which correspond to the two holes in a grooved wooden disk *B*. In the experiment the disk *B* prevents the inner cone from revolving when the spindle and the outer cone revolve; it is the friction between the two cones which converts into heat the mechanical energy

[1] This description is adapted, by permission, from the directions furnished by the manufacturers, W. G. Pye & Co., who in turn adapted it from the manuscript notes prepared by Mr. S. F. C. Searle for use in the Cavendish Laboratory, Cambridge, England.

supplied. A cast-iron ring, resting on the disk and fixed by two pins, serves to give a suitable pressure between the cones.

A brass wheel is fixed to the spindle, and by a string passing round this wheel and also round a handwheel motion is imparted to the spindle. A pair of guide pulleys prevents the string from running off the wheel. Above the wheel is a screw cut upon the spindle; this screw actuates a cogwheel of 100 teeth, which makes one revolution for each 100 revolutions of the spindle.

To the base of the apparatus one end of a bent steel rod is attached; the rod can be fixed in any position by a nut beneath the base. The other end of the rod carries a cradle in which runs a small guide pulley, this pulley being on the same level as the disk. The cradle turns freely about a vertical axis.

A fine string (*plaited* silk fishing line) is fastened to the disk and passes along the groove to its edge; it then passes over the pulley and is fastened to M, a mass of 200 or 300 g. On turning the handwheel it is easy to regulate the speed so that the friction between the cones just causes M to be supported at a nearly constant level.

To prevent the string from running off the guide pulley, a stiff wire with an eye is fixed to the cradle in such a manner that the eye is on the same level as the groove of the pulley and about 5 cm. from the axle of the pulley towards the disk. If the string be passed through this eye, it will always turn the cradle so that the string runs fairly over the pulley. In order to prevent the mass M from being wound up over the pulley, an eye is fixed to the steel rod, and the string supporting M passes through this eye. With these arrangements it is impossible, even with unskillful driving of the handwheel, either to throw the string off the guide pulley or to wind M up over the pulley.

Setting up the apparatus. Two tables will generally be required. The frictional machine, or "Jouler," as it may be called, is firmly clamped to one table, and the handwheel is clamped to the other table at a distance of ten or more feet. Care must be taken that the driving string runs properly, without any risk of slipping

off the handwheel. The steel rod is also fixed in a convenient position.

A thermometer is hung from a support so that it passes through the central aperture in the disk, and almost touches the bottom of the inner cone. The thermometer should also pass through the hole in the stirrer.

The string supporting M should be of such a length that, when as much as possible has been unwound from the disk, M is not quite in contact with the floor.

Before putting the cones together the rubbing surfaces must be carefully cleaned, and then four or five drops of oil must be put between them; the bearings of the spindle and guide pulleys should also be oiled.

Method of experimenting. The cones, cleaned and oiled, are weighed together with the stirrer. The inner cone is then filled up to about 1 cm. from its edge with water 2° or 3° below the temperature of the room, and the system is again weighed. The Jouler is now put into working order, one observer X taking his place at the handwheel, and a second observer Y at the Jouler. By working the Jouler the water is now warmed until its temperature is as nearly as possible equal to that of the room. After the index of the counting wheel has been read and the temperature (θ_1) of the water has been *carefully* observed and recorded, the operator X turns the handwheel fast enough to raise the mass M so far that the string supporting M is tangential to the edge of the disk. If the string be not tangential, the moment of its tension about the axis of revolution is seriously diminished. The observer Y stirs the water and notes the temperature at each passage of the zero of the counting wheel past the index; each passage of the zero after the first corresponds to 100 revolutions of the spindle. He gives a signal at each passage of the zero and X notes the time by aid of a watch. After Y has recorded the temperature upon a sheet of paper previously ruled for the purpose, he also records the time observed by X; unless these observations are methodically recorded, an error of 100 or 200 revolutions is quite probable.

Very accurate readings of these temperatures and times are difficult to make and are not necessary. It will be found that the time occupied by 100 revolutions of the spindle diminishes as the temperature rises; this effect is due to the diminution of the viscosity of the oil between the cones, consequent upon the rise of temperature. When M is 200 g. the temperature will rise about 1° C. for each 100 revolutions of the spindle in the case of the Joulers supplied by us.

After about 1000 revolutions have been made by the spindle of the Jouler, the motion is stopped and the highest temperature (θ_2) shown by the thermometer is carefully read. The index of the counting wheel is also observed, and from this reading, together with the initial reading and the number of complete revolutions made by the *counting wheel*, the exact number of revolutions (n) made by the *spindle* is ascertained.

Observations to determine the correction for cooling are then made. Without disturbing the apparatus, the temperature of the water is raised about 2° above θ_2 by working the Jouler. The Jouler is then brought to rest and the water is allowed to cool, with frequent stirring, and observations of the temperature are taken at intervals of about one minute till the water is only slightly (2° or so) above the temperature of the room.

Calculation of the correction for cooling. If there had been no loss of heat during the time that the Jouler was in action, the difference ($\theta_2 - \theta_1$) between the final and initial temperatures of the water could be used in the calculation without any correction. The correction necessary to allow for the loss of heat in the actual case is ascertained in the following manner. From the observations taken while the Jouler was in action a curve is plotted (the abscissa denoting time, and the ordinate temperature), showing how the temperature increased with the time. On account of the increase of speed due to the loss of viscosity of the oil the curve is concave upwards.

From the observations of the cooling of the water a second curve is drawn with time as abscissa and temperature as ordinate, and

from it the rate of cooling in degrees per minute at any particular temperature is determined from the slope of the tangent to the curve. It is best not to actually *draw* the tangent. If a triangular "drawing square" ABC be adjusted so that one of its edges AB touches the curve at the desired point, and if one of the other sides, as AC, be made to slide along a straight edge, AB can be moved parallel to itself until it passes through an intersection of the lines ruled on the squared paper. It is now easy to read off along the edge of the square the number of degrees corresponding to 10 minutes. Dividing this number by 10, the rate of cooling in *degrees per minute* is obtained for the particular temperature at which the tangent was taken.

A third curve is now constructed. From the first curve the temperatures of the water at the end of 1, 2, 3, $\cdots$ *minutes* are determined, and by the second curve the rates of cooling at these temperatures are determined. The third curve is drawn with the time as abscissa and the corresponding rate of cooling as ordinate. The origin is one point on the curve, since the water was initially at the temperature of the room. The area included between the line of times, the curve and the ordinate corresponding to the time when the Jouler was stopped, represents the correction to be added to $\theta_2 - \theta_1$, the observed rise of temperature. If 1 inch (or centimeter) on the squared paper correspond to p minutes, and 1 inch (or centimeter) correspond to q degrees per minute, then each square inch (or square centimeter) represents pq degrees. If the area be A square inches (or square centimeters), and ϕ be the correction to be added to $\theta_2 - \theta_1$, then $\phi = Apq$.

Since for small differences of temperature Newton's law of cooling is nearly true, and since the temperature of the water rises nearly uniformly the correction ϕ to be added to $\theta_2 - \theta_1$ is approximately equal to half the product of the rate of cooling at the temperature θ_2 and the time occupied by the rise $\theta_2 - \theta_1$.

Calculation of the work done. When the spindle has made n turns the work spent in overcoming the friction between the two cones is the same as would have been spent if the outer cone had

been fixed and the inner one had been made to revolve by the descent of the mass M grams. In the latter case M would have fallen through $2\pi nr$ cm., where r is the radius of the groove of the wooden disk. Hence the total work spent in overcoming friction and thus producing heat is $2\pi nrMg$ ergs.

Calculation of the heat produced. Let W grams be the mass of the water and w grams the mass of the cones and stirrer.

The specific heat of the metal may be taken as .095, and thus the system of cones, stirrer, and water is thermally equivalent to $(W + .095\,w)$ grams of water.

If the thermometer have a large bulb, it is necessary to take account of its water equivalent. The water equivalent is found by heating the thermometer and plunging it while hot into a small vessel of thin copper containing a known quantity of water at a known temperature. From the temperature of the thermometer at the instant when it is plunged into the water, and from the rise of temperature of the water, the water equivalent is calculated. Its value must be added to $W + .095\,w$ in the equation for J.

During the action of the Jouler the heat in the system of cones, stirrer, and water has increased by $(\theta_2 - \theta_1)(W + .095\,w)$ units, while $\phi\ (W + .095\,w)$ units have escaped from that system by radiation, conduction, and convection. Hence the total number of thermal units produced is $(W + .095\,w)(\theta_2 - \theta_1 + \phi)$ water-gram degrees.

Calculation of the mechanical equivalent of heat. If J denote the number of ergs of work which must be spent to raise the temperature of one gram of water by one degree, we have

$$J = \frac{2\pi nrMg}{(W + .095\,w)(\theta_2 - \theta_1 + \phi)}.$$

PART IV. EXPERIMENTS IN LIGHT

11. RANGE FINDING BY THE SEXTANT

The marine sextant has for its essential principles the so-called first law of geometrical optics: that the angle of reflection is equal to the angle of incidence, which has for its corollary that when a mirror is turned through a fixed angle, the incident ray remaining unchanged, the reflected ray is turned through twice the angle. As shown in the adjacent diagram, the sextant consists of a sector-shaped frame bearing a part (one sixth) of a graduated circle. Pivoted at the center of the circle, and at that point bearing a plane mirror, is an arm whose angular position can be read on the graduated circle. Attached to the frame is another plane mirror, but with only one half silvered; the other half, unsilvered, is transparent. Also attached to the frame and pointing to the half-silvered mirror is a telescope. In the sextant box there are usually three of these telescopes, one with a positive eyepiece, one with a negative eyepiece (practically an opera glass), one without any lens and practically little more than a peephole. Any one will do, but for most purposes the short opera glass serves best.

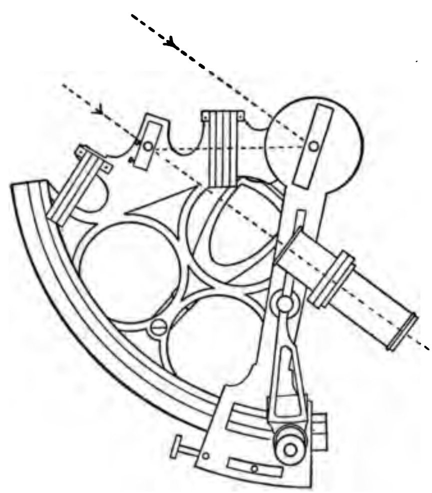

FIG. 19

In using the sextant it is important to bear in mind that the graduations on the circle, although marked as if whole degrees, are in reality half degrees. The object of this is to make the reading more direct in the determination of latitude and longitude,— the principal use of the sextant. In the determination of latitude at sea an observation is made of the angular altitude of the sun above the horizon. The instrument is first set so that the image of the sun seen by double reflection, i.e. from the movable mirror and the silvered portion of the stationary mirror, coincides with the image of the sun seen directly through the unsilvered portion of the stationary mirror. This is done by turning the movable mirror, and, although usually this second adjustment is not necessary or at most is very small, by turning the little capstan screw at the back of the fixed mirror. By this double adjustment, one at right angles to the other, exact coincidence of the two images can be secured. The corresponding reading of the graduated circle is what is called the zero reading of the instrument. The instrument is then directed toward the horizon and the movable mirror turned until the image of the sun seen by double reflection coincides with the horizon, and the reading of the graduated circle is again taken. The difference between the two readings is the angular altitude of the sun, which, subtracted from 90°, gives the latitude of the place, provided the observations are taken at the time of the equinoxes. At any other time of the year there must be applied a correction, taken from a table in the *Nautical Almanac*, for the angle of the sun above or below the plane of the equator. On land an artificial horizon must be used and the angle measured is the angle between the sun and its image seen by reflection in a liquid surface. But while latitude and longitude determinations are the more common uses of the instrument, the object of the present experiment is range finding.

Before the modern range finders were perfected the use of sextants for this purpose was not uncommon. Given the height of the top of the smokestack of a war vessel above the water line, the distance of the vessel is determinable with considerable accuracy

by means of the sextant. Looking through the sextant at the vessel, its doubly reflected image may be made first to stand on top of the image seen direct, touching the water line to the top of the smokestack, and then beneath, touching the smokestack to the water line. From the difference between the two readings and the height of the smokestack above the water line, by very simple triangulation, the distance of the vessel can be calculated.

Place across the room a meter rod and find its distance by means of the sextant. In the calculation bear in mind that the graduations on the divided circle are half degrees, that on turning a mirror through an angle the reflected beam is turned through twice the angle, and that the reflected image has been turned through twice the angle subtended by the meter rod. In this work it will probably be necessary to make a negative reading of the graduated circle, but a little consideration will show how this should be done.

After calculating the distance, measure it by means of a tape line. While working with the sextant note carefully the position of the pivot and measure to this point with the tape. It is desirable to so work that this point is pretty well fixed, for the experiment is capable of a surprising degree of accuracy.

12. PHOTOMETRY

There are many types of photometers of which the most common commercial forms depend upon the law of the inverse square of the distance. The two sources of light which are to be compared, the standard and the light under investigation, are placed at a considerable distance apart, and a screen moved until it is equally illuminated by each. The relative intensities of the two sources are then proportional to the squares of their distances from the screen. Of this type the Lummer-Brodhun form is a very satisfactory example. In this photometer the two sources of light are placed at opposite ends of a long scale on

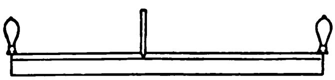

FIG. 20

which slides the so-called "head" of the photometer. The head of the Lummer-Brodhun photometer is shown in the adjacent diagram. It consists of a white plaster of Paris disk S, both sides of which are viewed at the same time through the telescope T. This is accomplished by means of a system of mirrors, two of which are shown at M and M'. The light from the mirror M is reflected by the right-angle prism P into the telescope by total internal reflection from the back of the prism. There is a corresponding prism P'. The two prisms P and P' do not touch except over a certain area where they are cemented together by Canada balsam. Where the prisms do not touch the light is entirely reflected by the prism P, and all the light received is from the mirror M and the side S of the screen. Where the prisms are cemented together by the Canada balsam no light is reflected, but the light is entirely transmitted from the prism P', and the light comes from the mirror M' and the side S' of the screen. The telescope is focused on the surface of separation of the two prisms, and the observer, thus seeing side by side the two surfaces S and S', can see when the two sides are equally illuminated. The head of the photometer is moved backward and forward until this is accomplished. The position of the head on the long scale is then read and the relative intensities of the two sources calculated by means of the ratio of the squares of the distances.

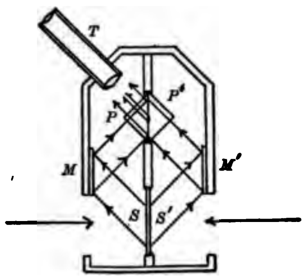

FIG. 21

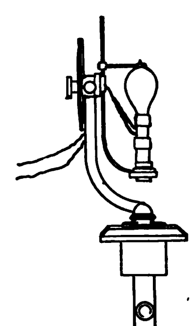
FIG. 22

Most sources of light, for example a fish-tail gas jet or an incandescent electric lamp, have different intensities in different directions. It is customary in commercial photometric work to make a measurement of the candle power in different directions. The light under examination is so

arranged that it can be rotated in all directions. Usually two planes are taken, one transverse and the other a vertical plane at right angles to the first. The candle power of the flame or incandescent lamp is then measured at different angles in the horizontal plane. It is then plotted on a diagram similar to that shown in the adjacent figure. A corresponding experiment is carried out in the plane at right angles, and a corresponding chart is made. With an incandescent lamp this double orientation is not difficult. With a gas jet the orientation in a horizontal plane is not difficult, but the investigation of the candle power in various directions in the vertical plane is more difficult. It is usually accomplished by means of mirrors.

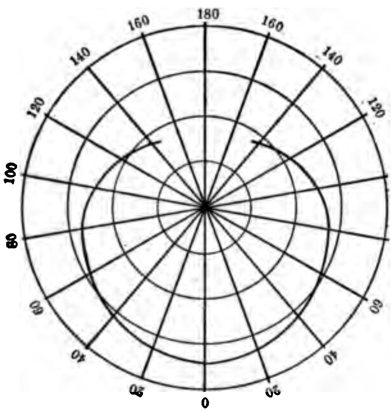

FIG. 23

The unit in terms of which light is usually measured is that of a paraffin candle burning at the rate of 120 grains, 7.78 g., per hour. It is, however, difficult to get a paraffin candle to burn steadily, and it is customary to use a secondary standard which has at some time been calibrated by comparison with a standard candle.

Determine and plot the curve in both a horizontal and a vertical plane of an incandescent lamp, using as a standard another incandescent lamp fixed so that it cannot rotate at the other end of the photometer bench, and fed from the same electric-light circuit. The experiment may, of course, be varied by using different forms of incandescent lamps, or by using with the lamps different forms of reflectors.

Or, determine and plot the curve in a horizontal plane of a fish-tail burner, comparing it with another fish-tail burner, which is of course not to be rotated, at the other end of the bench.

13. SPECTROMETER

ANGLES OF A PRISM

First Method

Of the two telescopes belonging to a spectrometer, one, called the collimator, is for rendering parallel the rays of light; the other is the observing telescope. The following are the adjustments necessary. Focus the cross hairs in the observing telescope until they can be seen distinctly without straining the eyes. Focus the telescope on some distant object, adjusting until it can be seen sharply with the cross hairs, and until there is no parallax between them. Turn both telescopes until they point directly toward the center of the instrument, and clamp firmly. Place at a little distance in front of the slit a Bunsen burner (or alcohol) flame colored by common salt or by carbonate of sodium. Darkening the room slightly, turn the observing telescope about the central axis until it is opposite the collimator, and alter the length of the latter until the image of the slit as seen in the observing telescope is in sharp focus and without parallax with the cross hairs. Level, by means of a spirit level, the divided circle, the collimator, and the observing telescope. The slit in the collimating telescope should be turned quite vertical. If a prism is to be used with the spectrometer, its edge should also be vertical. To adjust this, before placing the prism on the central stand, view the slit direct, the two telescopes being opposite each other. If the two telescopes are at the same height, the image of the slit should be nearly in the center of the field of view. Note its position vertically, with

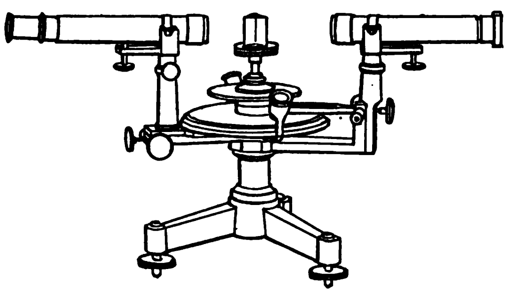

FIG. 24

reference to the top and bottom of the field of view or with reference to the horizontal cross hair. Place the prism on the stand with one edge turned toward the collimator, and turn the observing telescope until the image of the slit can be seen by reflection from one face of the prism. Level the prism until the image of the slit occupies the same part of the field of view as when seen directly. Repeat when viewing the slit reflected from the other face, and then, turning the telescope back, see that the result of the first leveling has not been disturbed. The slit in the collimator should be narrow. All these adjustments are generally necessary. The graduated circle, as the most delicate and valuable part of the instrument, should be treated with special care.

A spectrometer may be used as a goniometer in measuring the angles of a prism.

Place the prism at the center of the spectrometer with one edge turned toward the collimator. Set the cross hairs of the observing telescope upon the image of the slit reflected from one face of the prism, and read the graduated circle. Turn the telescope until the cross hairs are set upon the image of the slit reflected from the other face. Measure the angle through which the telescope has been turned. It will be readily seen by an inspection of the figure that this angle is twice that of the prism. Having lettered the different angles of the prism so that they can be readily identified, measure all three angles of the prism. Their sum should equal 180°. The divided circle may now be unclamped, turned quarter round, and the work repeated, thus correcting in part for errors of graduation. Care must be taken not to confuse the reflected light with the refracted light which has passed around through the glass.

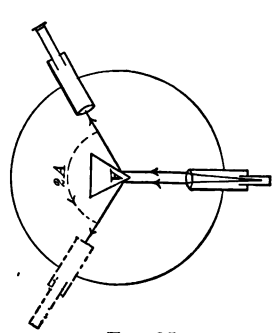

FIG. 25

Throughout the experiment neither telescope should be touched after the preliminary adjustment. In turning the observing telescope hold as near the solid bearing as possible.

Angles of a Prism

Second Method

Clamp the observing telescope permanently near the collimating telescope. Place the prism upon the leveling base so that the image of the slit will be thrown into the observing telescope. Without disturbing either telescope turn the prism by means of the tangent screw until the image coincides with the cross hair. Take the reading on the graduated circle. Turn the prism until the image of the slit is reflected from the next face. Adjust accurately by the tangent screw and again read. The angle through which the prism has been moved, subtracted from 180°, will be the angle A of the prism turned past the telescope. Repeat, using a part of the circle differing by 90° from the preceding. Take the mean. In this way measure the angle of the prism.

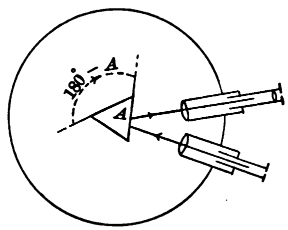

Fig. 26

Index of Refraction

When a ray of light passes from a rare to a denser medium it is bent toward the perpendicular to the surface. For the same substances and the same colored light the sine of the angle of incidence divided by the sine of the angle of refraction is a constant. It is called the index of refraction, and is usually denoted by the letter n. The index of refraction of light passing from air to glass may be most conveniently determined, when the glass is in the form of a prism, as follows.

Having completed the adjustments outlined above, turn the prism into the position indicated in Fig. 27. Set the observing telescope upon the refracted image. Rotate the prism, following the refracted ray with the observing telescope. A position can be

found at which the light is least refracted. Clamp the support of the prism. Set the cross hairs on the image of the slit and read the position of the telescope. Remove the prism and set the telescope upon the image of the slit seen directly. The angle through which the telescope has been turned is the angle of deviation of the light, in this case the angle of minimum deviation D.

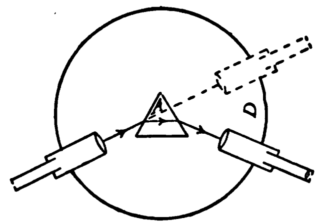

FIG. 27

In the position of minimum deviation the light passes through the prism in a direction such that it makes equal angles with the surface of the glass in entering and in leaving.

$$i = i_1 \quad \text{and} \quad r = r_1.$$

Consider the small triangle formed by the three paths of the ray extended. From the property of the exterior angle of a triangle,

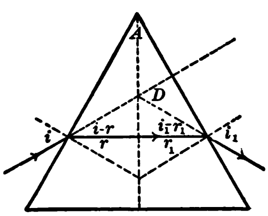

FIG. 28

$$D = (i - r) + (i_1 - r_1) = 2\,i - 2\,r.$$

From the property of similar triangles each half of A is equal to r.

$$A = 2\,r \quad \text{or} \quad r = \frac{A}{2},$$

and substituting

$$D = 2\,i - A,$$

$$i = \frac{D + A}{2}.$$

Since

$$n = \frac{\sin i}{\sin r},$$

$$n = \frac{\sin\left(\dfrac{D + A}{2}\right)}{\sin\left(\dfrac{A}{2}\right)}.$$

The value of A may be taken from the previous experiments.

Measure the index of refraction of the glass for the yellow light of sodium and the red light of lithium, using in each case different parts of the divided circle.

14. WAVE LENGTH OF LIGHT BY NEWTON'S RINGS

The wave length of light was first measured by Newton, who did not recognize, however, that the distances measured were wave lengths, but, having propounded the Corpuscular Theory of Light, maintained that the distances were the lengths of "fits of easy reflection and refraction." A hundred years later the true significance of his measurements was recognized. His investigation arose from a study of the color of thin plates, such, for example, as the colors of soap films, oil on water, oxide films on tempered steel, etc. To make his conditions more determinate he investigated the colors formed by a convex lens resting against a plain glass surface. The conditions being symmetrical about the point of contact, the colors were disposed in concentric rings. The phenomenon thus produced has since been called Newton's rings.

Whenever light meets a surface separating two media of different indices of refraction a portion is reflected. Thus in the arrangement indicated in the adjacent diagram, light is reflected from the front surface of the lens and from its back surface, from the front surface of the plate glass and from its back surface. With that reflected from the front surface of the lens and from the back surface of the plate glass we are not here concerned; but the light reflected from the back of the lens and from the front of the plate glass have traveled but slightly different distances and are in a condition to interfere. In so doing they will either reënforce each other, or mutually destroy each

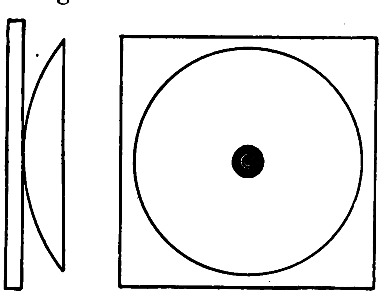

Fig. 29

other, according as they are in the same phase or in opposite phases. For a moment consider only difference in path. At the point of contact there is no difference in path, and the two portions of light should reënforce each other. At such a distance from the point of contact that the thickness of the air film is $\frac{1}{4}$ wave length, the difference in path, across and back, is $\frac{1}{2}$ wave length, and, considering path difference only, the two waves should be in opposite phases and cancel each other. There should, therefore, at this distance from the point of contact be a dark ring. When the distance from lens to plate glass is $\frac{1}{2}$ wave length the difference in path is a whole wave length, and, again considering only difference of path, there should be a bright ring. There is, however, another consideration that exactly reverses the above conclusions. One reflection is of light passing from glass to air, the other is in passing from air to glass; one is with half a wave change of phase, the other without change of phase in the reflection. This reversal of one portion of the light results in an exact reversal of the bright and dark rings, so that the center is dark, the first bright ring occurs where the distance from lens to plate is $\frac{1}{4}$ wave length, the first dark ring where the distance from lens to plate is $\frac{1}{2}$ wave length, etc.

Place the plate glass against the lens, and illuminating the combination by the yellow light of the sodium flame, measure, by means of the cathetometer, the diameters of a large number of the rings. From a knowledge of the radii of the rings and the radius of curvature of the lens, calculate for each of the several rings the corresponding thickness of the air film. The necessary geometrical formula is easily deduced, — $d = \dfrac{r^2}{2R}\cdot$ From these results determine the wave length of sodium light. Similarly, determine the wave length of lithium light.

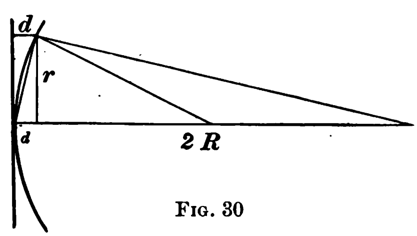

FIG. 30

In measuring in this manner such a small magnitude as the wave length of light, it is obvious that the greatest delicacy of manipulation is necessary. The least dust at the point of contact, or, on the other hand, excess of pressure and consequent flattening of the glass, vitiates the work. It is perhaps hardly necessary to add that the lens and plate glass must be turned in such a position as to reflect the light from the flame into the telescope of the cathetometer, and that the latter should be focused on the surface of the lens and plate glass.

15. WAVE LENGTH OF LIGHT BY THE DIFFRACTION GRATING

A better method of measuring the wave length is by means of the diffraction grating mounted on a spectrometer. The method was devised by Fraunhofer, who at first used a grating made of parallel wires. Crude as the device was, his determinations of the wave length of sodium light was surprisingly accurate. The modern diffraction grating is made by diamond rulings on glass or speculum metal.

The arrangement of the apparatus for a transmission grating on glass is shown in the adjacent diagram. The series of dots between the collimating telescope and the observing telescope indicates the grating seen in cross section. The grating is placed on the central stand, with its plane at right angles to the axis of the collimating telescope and with its rulings vertical. A series of spectra are thus produced, lying both to the right and to the left of the direct image of the slit. The explanation of the formation of these spectra is as follows:

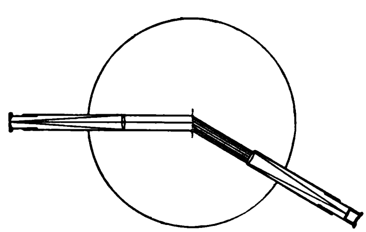

FIG. 31

The light passing through the narrow space between the rulings spreads laterally, but is of feeble intensity except in such

directions that the light coming from all the openings are in phase with each other, and therefore additive in their total effect. Thus, in the diagram, if the angle with the original direction of the light is such that the path of each ray is longer than the one next to the left of it by exactly one wave length, the light coming from the several openings will be in the same phase. In this direction the light will be intense and comparable with, though not equal in intensity to the light seen directly through the grating. The same would be true with a difference of path of two, three, or any whole number of wave lengths. In the adjacent diagram, which represents a few openings on an enlarged scale, l is obviously equal to the product of the distance between the rulings, d, multiplied by the sine of the angle a, the angle between the image of the slit seen directly and the diffracted image.

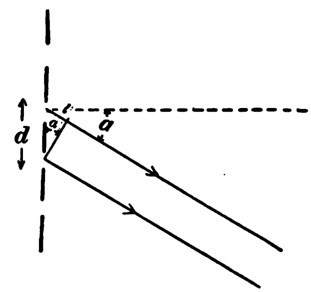

FIG. 32

Measure the angle of diffraction of the first and second images of the slit, both to the right and to the left, and calculate the wave length of sodium light. Note that the sodium line, the so-called D line, is double, and measure the wave length of each, the D_1 and D_2 lines as they are separately called. Similarly, measure the wave length of the lithium red line.

16. THE DIRECT-VISION SPECTROSCOPE

The spectrometer is provided with a graduated circle and is designed primarily for accurate quantitative work in the measurement of wave lengths or indices of refraction; while the spectroscope is designed only for a qualitative examination and comparison of spectra and is without a graduated circle. In the following work a direct-vision spectroscope will be used. It consists of a train of prisms, alternately of flint and crown glass, so placed as to give dispersion of the light into a spectrum without considerable

refraction of its center. The dispersion of the instrument is rather more than that of the spectrometer used above. It also has certain other conveniences; for example, by means of a small auxiliary tube an illuminated scale may be reflected into the field of view,

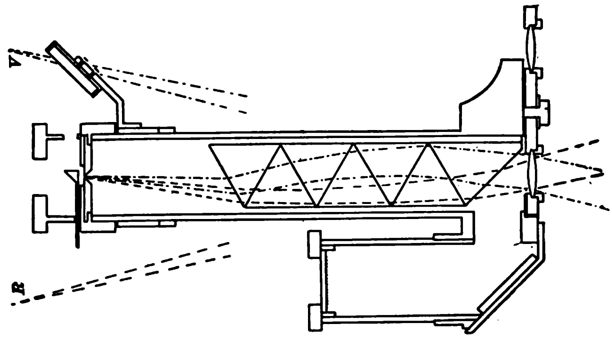

FIG. 33

and by means of it the relative positions of lines may be determined. Further, a small right-angle prism, which may be thrown by a little lever in front of one half of the slit, makes it possible to obtain in the field of view, at the same time and side by side, spectra of two different sources of light, and therefore to make direct comparison. It is customary to plot spectra in the manner shown in the adjacent diagram, where the scale may be either wave lengths or entirely arbitrary.

Plot the spectra of sodium, lithium, barium, and strontium, getting their light by burning some easily volatilized salt in the Bunsen burner flame.

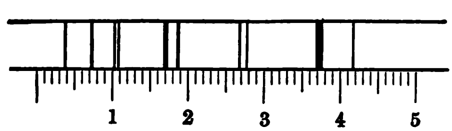

FIG. 34

Observe also the spectrum of sunlight. For this purpose it will be sufficient to turn the slit of the spectroscope toward the sky.

Compare the spectrum of sunlight with the spectrum of the sodium flame. This may be done by turning the slit of the

spectroscope toward the sky and, by means of the little right-angle prism which can be thrown in front of the slit, reflecting the sodium flame into the slit. Note the coincidence of the sodium line with a dark line in the solar spectrum.

The spectra of gases is generally secured by passing the spark from the secondary of an induction coil through a tube containing the gas at a low pressure. The tube is generally constricted for a considerable length in order to secure concentration of light, and the constricted part is viewed end on. Observe and plot the principal characteristics of the spectrum of hydrogen or argon or helium.

The following part of the experiment is difficult and should be attempted only by those especially interested in the subject.

By means of elements, the wave lengths of whose lines are known (being taken from tables) calibrate the arbitrary scale in the direct-vision spectroscope. Thus determine the reading on the scale corresponding to certain wave lengths well distributed throughout its length. Plot a curve having scale readings for abscissæ and wave lengths for ordinates. This spectroscope thus calibrated may be used to measure wave lengths.

Observe the spectrum of some unknown substance. Determine from the curve the wave length of its principal lines, and their relative intensity. Comparing these measurements with tables of wave lengths, determine the unknown substance. In this experiment, difficult at best, use as the unknown substance some single element, simple salt. If the unknown substance is a complex mixture, the method of analysis is very difficult and only possible when care is taken to secure very great accuracy in the measurements.

17. NATURAL ROTATION OF PLANE POLARIZED LIGHT

The two principal methods of producing polarized light are by reflection from glass and by means of a Nicol prism. When light is reflected from ordinary glass at an angle of about 55° the reflected light is totally plane polarized, the portion transmitted

being partially plane polarized. In order to increase the amount of light reflected, a pile of thin glass plates is often used. The transmitted light is thereby reduced, but it is more completely polarized. The Nicol prism is made of a rhomb of Iceland spar, cut in two at a suitable angle and recemented with Canada balsam. The Nicol prism is much the better of the two, but is also the more expensive.

One of the interesting phenomena of polarized light, often made use of commercially, is shown by sugar solutions and by many of the essential oils. When polarized light is passed through a solution of sugar in a long glass tube the plane of polarization is rotated, the rotation being proportional to the length of the

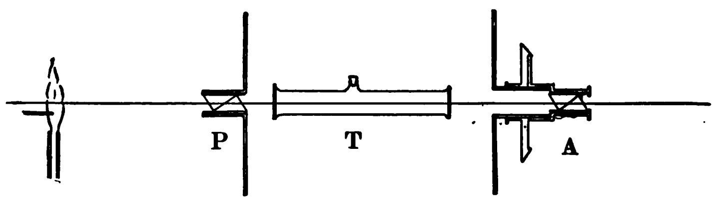

FIG. 35

column of liquid and to the strength of the solution. The following arrangement will show this phenomenon of the rotation of the plane of the polarization. The Nicol prism, as shown in the adjacent diagram, is mounted on the stand *P*. This prism is ordinarily called the polarizer. Another Nicol prism mounted on a second stand can be rotated and its angle of rotation read by means of a small graduated circle. This second prism is called the analyzer. Between the two is a tube *T* containing the liquid whose rotating power is to be measured. At some distance in front of the polarizer is placed a Bunsen burner flame burning sodium, and all the parts are so aligned that the light will pass through the whole series. The adjustment should be begun by removing the tube *T*. Looking through the analyzer and the polarizer toward the flame, so turn the analyzer that it will extinguish the light which has passed through the polarizer.

Note the reading on the graduated circle on which the analyzer is mounted. Insert the tube containing the sugar solution, and the plane of polarization of the light transmitted by the polarizer will be found so turned that the analyzer will no longer extinguish it. Turn the analyzer until the light is again extinguished. Reread the divided circle. The difference in the two readings is the angle through which the plane of polarization has been turned by the sugar solution. There is one element of ambiguity in this experiment. There is nothing to show whether the plane of polarization has been rotated to the right or to the left, nor whether it may not have been turned several times around. To remove this ambiguity, the tube may be filled with the same solution diluted to half its original strength. The rotation of the plane of polarization by this solution will be half that of the other. A moment's consideration of these two results will show whether the rotation has been to the right or to the left, and whether the rotation has been less or greater than 180°.

In this way measure the rotation of the plane of polarization of the sugar solution and calculate from both results the amount of rotation per centimeter of length of a one per cent solution. Determine also the direction of rotation.

The rotation constant having been determined for sugar, determine the concentration of a solution of unknown strength by measurement of its rotating power.

The rotation is different for different colors of light, and the above result gives the rotating power of the sugar solution for sodium light, which is usually taken as standard. An interesting variation of this experiment is as follows: The rest of the apparatus remaining as before, i.e. with the polarizer, the sugar solution and the analyzer in line, substitute for the Bunsen burner flame the flame of a kerosene lamp or a gas jet, and instead of looking through the analyzer with the naked eye, place between the eye and the analyzer a small direct-vision spectroscope. The spectrum will now be traversed by a dark band, or, if the rotation is sufficiently great, by several dark bands. On rotating the analyzer

this band or bands will move through the spectrum. The position of the center of the dark band in the spectrum corresponds to that particular wave length of light which has been rotated by the sugar solution through the angle through which the analyzing prism has been turned.

By an arrangement of apparatus of this sort and by an extension of this reasoning, determine the relative rotating power of the sugar solution for red, yellow, green, and blue light. Strict work would, of course, require that one should determine the rotating power for a particular wave length of light, but it is sufficient for the present purpose simply to determine it for the middle of these several colors.

Determine also in this way the rotating power per centimeter of length of a quartz crystal, so cut that the light can pass through parallel to the axis of crystallization. Determine the rotating power of the quartz for each color of the spectrum.

Plot the results obtained with the sugar solution and with the quartz, plotting the rotating power as ordinates against the wave length of the middle of the several colors as abscissæ.

18. EQUIVALENT FOCAL LENGTH OF COMPOUND LENSES

In the theory of simple lenses it is ordinarily assumed that the thickness of the lens is negligible in comparison with the other distances involved, such, for example, as the radii of curvature and the principal focal length. In such cases we obtained the well-known formula

$$\frac{1}{p}+\frac{1}{q}=(n-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)=\frac{1}{f},$$

where q is the distance to the image, p the distance to the objective, n the index of refraction, R_1 and R_2 the radii of curvature, and f the principal focal length of the lens. These distances may all be regarded as measured to the center of the lens. The above

formula may be proved very readily either by the conception of rays and the ordinary processes of geometrical optics, or (and very much better) by the method of waves, — a method of analysis more rigorous and more effective.

When the lens is a thick lens, or when, as in the case of the ordinary microscopic objective, it is composed of several lenses, the above formula is not so readily applied; nevertheless we have a quantity which we ordinarily call the equivalent focal length, meaning by that the focal length of a thin lens which would produce an equal magnification, the total distance from image to object being the same. This is illustrated in the adjacent diagram.

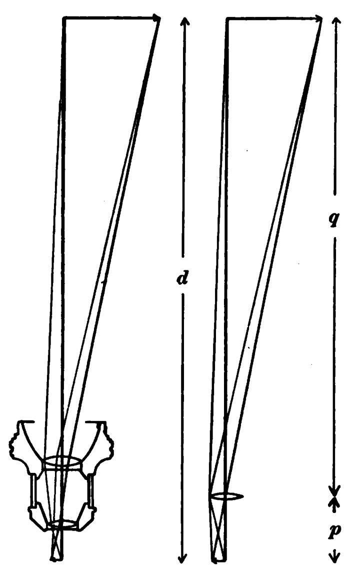

Fig. 36

In a compound microscope the ratio of the size of the real image formed by the objective to the size of the object is spoken of as the initial magnification by the objective. In the compound microscope this real image is then magnified by the eyepiece. The object of the present experiment is to determine separately the equivalent focal length of the objective and of the eyepiece of such a compound microscope. Place on the stage of the microscope a slide ruled into tenths and hundredths of millimeters, called ordinarily a stage micrometer. Instead of the ordinary microscopic eyepiece use a micrometer eyepiece, having as its essential characteristic a diamond scratch in the field of view which can be moved transversely by means of a micrometer screw. Measure by means of this micrometer screw the magnified image of the stage micrometer. The ratio M of the size of the image to the size of the original scale determines the magnification produced. This initial

magnification being determined, we may proceed in the following way to determine the equivalent focal length of the objective. Measure the distance from the stage micrometer to the plane of the diamond scratch in the micrometer eyepiece. This distance being great, it can be determined with a fair percentage of accuracy without excessive difficulty. If we call this distance d, the two following equations immediately hold.

$$p + q = d,$$

$$\frac{q}{p} = M.$$

Calculate by this data and by a solution of these two formulæ the value of p and q of the equivalent simple lens, and substituting these values of p and q in the formula

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f},$$

calculate the value of f, the equivalent focal length of the objective.

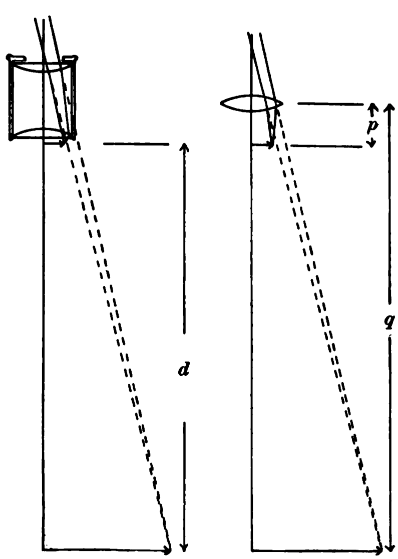

FIG. 37

In order to determine the equivalent focal length of the eyepiece, proceed as follows: Leaving the stage micrometer in position, the same eyepiece, in fact, the whole adjustment just as it has been, place on the stage of the microscope, but to one side of the instrument, a small millimeter scale. Focus the microscope so that it is possible to see with comfort one scale through the microscope and the other scale with the unaided eye. Proceed as in determining the magnification produced by a telescope, to determine the magnification produced by the compound microscope. The initial

magnification produced by the objective having already been determined, it is obviously possible to determine from this result the magnification M', produced by the eyepiece alone. We have the formulæ, indicating by primes the eyepiece values,

$$\frac{q'}{p'} = M',$$

$$q' - p' = d;$$

and since the image formed by the eyepiece is virtual,

$$\frac{1}{p'} - \frac{1}{q'} = \frac{1}{f'}.$$

Determine the value of f'.

PART V. EXPERIMENTS IN MAGNETISM AND ELECTRICITY

19. HORIZONTAL COMPONENT OF THE EARTH'S MAGNETIC FORCE

MAGNETIC PENDULUM

The direction of the force which the earth exerts on a north-seeking pole is, in general, not horizontal but more or less dipping. However, most magnetic and electro-magnetic instruments are so constructed that we are concerned only with the horizontal component of this force. The horizontal force exerted on a pole of unit strength is denoted usually by the letter H. If a magnet which can turn about a vertical axis is set east and west, the force on each pole tending to turn the magnet is HQ, Q being the strength of each pole and H the force on each unit. If the distance from the axis to each pole is d, the turning moment of each force is HQd. Since the two forces tend to turn the magnet in the same direction, the total moment is $H\,2\,Qd$.

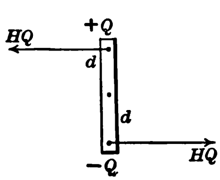

FIG. 38

The factor $2\,Qd$ is the moment of the force which would be exerted on the magnet if it were placed in, and at right angles to, a magnetic field of unit strength. It is called the magnetic moment of the magnet, and is usually denoted by the letter M. The moment of the directive force of the earth's field on the magnet may therefore be written MH. If a suspended magnet be displaced *slightly* from its normal north and south direction and then released, it will oscillate to and fro. By analogy with the formulæ of Experiment 6, it may be shown that if t denotes the time

of oscillation and I the moment of inertia of the magnet and support,

$$t = \pi \sqrt{\frac{I}{MH}}.$$

Place a short bar magnet centrally upon a light support, and suspend the latter by a small strand of silk from which all initial twist has been removed by allowing it to untwist with an equal but nonmagnetic weight attached. Turn this magnet and support through an angle of about 5°, and allow the combination to oscillate. Time very carefully

$$t_1 = \pi \sqrt{\frac{I_1}{MH}}.$$

Here I_1 is the moment of inertia of the oscillating mass, — the magnet and support. Place in position a brass ring and again time, the ring being so placed that it oscillates about its center

$$t_2 = \pi \sqrt{\frac{I_1 + I_2}{MH}}.$$

I_2 is the moment of inertia of the ring. The value of the directive moment MH remains the same. Following the method of Experiment 6, eliminate, and find I_1. Substituting this value of I_1 in the first equation, find the value of the product MH. When reduced for this purpose the equation becomes

$$MH = \pi^2 \frac{I_1}{t_1^2}.$$

Throughout the experiment avoid jarring or heating the magnet, or bringing it into contact with other magnets or pieces of iron. Rough handling might vary its magnetization. Preserve the magnet with the same care during the second half of the experiment.

By the present experiment the product MH has been determined. In the next experiment the ratio $\frac{M}{H}$ will be found, and by eliminating between the two, the values of M and H may be found separately. Of the two, H is the more important.

THE MAGNETOMETER

The ratio $\frac{M}{H}$ may be found by the deflection which a bar magnet will give a small suspended magnetic needle. The apparatus for this experiment is called a magnetometer. At the center of the instrument is suspended a magnetic needle, the comparatively small deflections of which may be read by means of a mirror, telescope, and scale. A long arm on each side of the instrument acts as a support, on which may be placed the bar magnet of the last experiment.

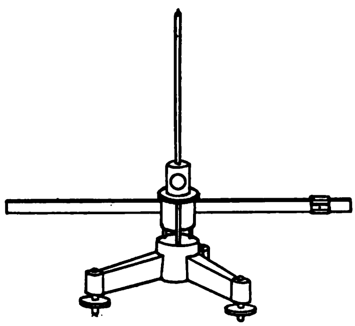

FIG. 39

In measuring the angle of deflection by means of the mirror telescope and scale it is to be remembered that the deflection read on the scale divided by the distance from the mirror to the center of the scale is the tangent of twice the angle through which the needle is deflected.

Set the apparatus so that the needle will be at the place at which the magnet was suspended in the last experiment. Turn the supporting arm east and west, level the instrument, and adjust the telescope and scale to read zero. Place the magnet of the last experiment on the support, with its center at a distance r east of the needle, and with its axis east and west. Read the deflection of the needle. Turn the magnet end for end and again read. Place the magnet at the same distance to the west and repeat. Call the average of these angles a. The force on the north pole of the small compass needle is, if we call q its pole strength, $\frac{Qq}{(r-d)^2}$ attracting,

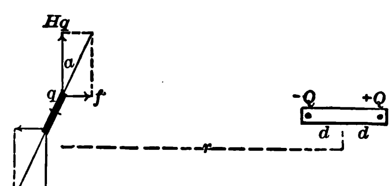

FIG. 40

and $\dfrac{Qq}{(r+d)^2}$ repelling, and the resultant force

$$f = \frac{Qq}{(r-d)^2} - \frac{Qq}{(r+d)^2}, \quad \text{or} \quad f = \frac{4\,rdQq}{(r^2-d^2)^2}.$$

We may consider d as being so small that its square may be neglected in comparison with the square of r,

$$f = \frac{4\,rdQq}{r^4} = \frac{2\,Mq}{r^3}.$$

The pole is also drawn north by a force Hq. These forces are represented in the adjacent diagram. Since the needle is free to rotate, it will set itself with its axis in the direction of the resultant force. Hq and f are at right angles to each other, therefore

$$\frac{f}{Hq} = \tan a.$$

A similar consideration of the forces acting on the south pole of the needle shows that the needle is also in equilibrium as regards these forces when deflected through this angle a.

Substitute for f its value $\dfrac{2\,Mq}{r^3}$,

$$\frac{2\,Mq}{r^3 Hq} = \tan a;$$

whence

$$\frac{M}{H} = \frac{1}{2} r^3 \tan a.$$

Calculate the value of $\dfrac{M}{H}$. In the last experiment the value of the product MH was determined. Eliminate and find H. The result will be in dynes. Eliminate in a different way and find M.

The experiment should be repeated with the magnet placed at different distances from the suspended needle.

The neighborhood of knives, window weights, and other movable magnets should be avoided.

GALVANOMETERS AND AMMETERS

The following is not an outline of an experiment, but is a discussion of galvanometers and ammeters preliminary to their use in the subsequent experiments.

There are several kinds of instruments for measuring the strength of an electrical current. Of these the tangent galvanometer measures directly in absolute units, but is only adapted to the measurement of comparatively strong currents. The astatic needle galvanometer is sensitive to exceedingly small currents, but measures them in either absolute or practical units only after it has been standardized by comparison with a tangent galvanometer, or its equivalent. The D'Arsonval galvanometer is also sensitive to exceedingly small currents, and for some work possesses very great advantages over the astatic needle galvanometer. The ordinary commercial instruments for measuring strong currents are called ammeters, and for small currents are called milliammeters. Both instruments are graduated to read the strength of the current directly, this standardization having been made by comparison with some absolute instrument, such, for example, as the tangent galvanometer.

As these instruments will be used continually in the following experiments, a description of their essential parts is here given.

TANGENT GALVANOMETER

When a current of electricity flows through a straight wire it is surrounded by a magnetic field in which the lines of force are concentric circles. Thus the force on a positive magnetic pole would be such as to carry it in a circle around the wire in a direction corresponding to the motion of the hands of a watch facing an observer looking along the wire in the direction of the current. The absolute unit current of electricity is such that if flowing through

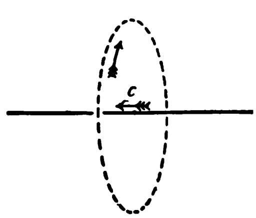

FIG. 41

a wire one centimeter long, bent in the arc of a circle whose radius is one centimeter, it will exert a force of one dyne on the unit pole placed at the center of the circle. If the length of the wire be l, the force will be l times as great; if the radius of curvature be r, the force will be $\frac{l}{r^2}$, since the force varies inversely as the square of the distance; if the wire make a complete circle, the force will be $\frac{2\pi r}{r^2}$; and if there be n turns of wire, the force will be $\frac{2\pi rn}{r^2}$, or $\frac{2\pi n}{r}$. This, the force exerted at the center of a coil when the unit current flows through it, is called the constant of the coil, and we shall denote it by the letter G. When a current C flows through the coil the force is C times as great, and if the magnetic pole is of strength q, the force is GCq, acting at right angles to the plane of the coil.

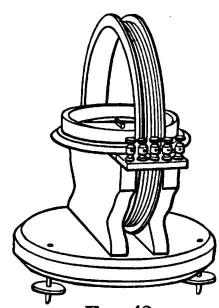

FIG. 42

If a thin coil of large radius is placed vertically in a north and south plane, and a small magnetic needle hung at its center, the instrument constitutes a tangent galvanometer. Referring to the diagram, and employing the reasoning of the last experiment, the force drawing the north pole of the needle north is Hq, and that drawing it east (or west, as the case may be) is the force f, equal to GCq. Since the needle is free to rotate, it will turn into the direction of the resultant force, making an angle of deflection, a, such that

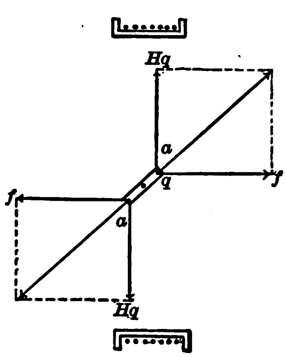

FIG. 43

$$\frac{f}{Hq} = \tan a, \quad \text{or} \quad \frac{GCq}{Hq} = \tan a;$$

whence $$C = \frac{H}{G}\tan a,$$

or $$C = \frac{Hr}{2\pi n}\tan a \left\{\begin{array}{l}\text{absolute}\\ \text{units.}\end{array}\right.$$

By means of this formula the current C may be calculated in absolute units.

The practical unit of current is called the ampere, and is equal to one tenth of an absolute unit. The formula, therefore, for calculating the current in amperes is

$$C = \left(10 \frac{Hr}{2\pi n}\right) \tan a.$$

The part in parenthesis is called the reduction factor of the galvanometer and will be denoted by the letter F.

On setting up a tangent galvanometer, the first and most essential adjustment is so to place the instrument that the needle shall lie in the plane of the coil when no current is flowing. After having made this adjustment as nearly as possible by eye, complete the adjustment by the following electrical method. Turn the compass until the pointer on the needle reads zero. Pass a current from a single Daniell cell through the reversing key and through the coil of the tangent galvanometer, using a number of turns of wire n, such that the deflection will be as near 45° as possible. Take the reading of both ends of the pointer. Reverse the current and again read. If the deflection is not the same in the two cases, the coil is not in the north and south plane. An inspection of the adjacent diagram will show that the coil is to be turned toward the needle (not the pointer) when making the smaller deflection. Each time the coil is turned the compass must be readjusted so that the pointer of the needle will read zero before the equality of the deflections is again tested.

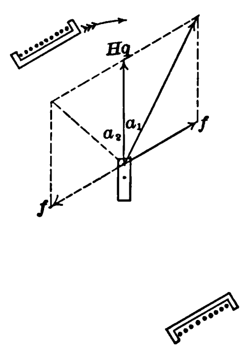

FIG. 44

In using a tangent galvanometer, the experiment should be so arranged, if possible, that the deflection will be in the neighborhood of 45° — between 30° and 60°. This may generally be accomplished by varying n, the number of turns of wire. Both

ends of the pointer should always be read to correct for possible eccentricity. The wires leading to and from the tangent galvanometer ought always to be twisted about each other, that they may exert no direct influence on the needle. Whenever the experiment permits a reversing key should be inserted in the tangent galvanometer circuit, and deflections taken in both directions. If the needle is balanced on a pivot point, the galvanometer should be tapped gently before reading. The jarring will assist the needle in settling to its correct position, overcoming the friction of the pivot. When not in use the needle should be lifted from the pivot.

Astatic Needle Galvanometer

The galvanometer may be made more sensitive either by so winding the coil as to increase the force exerted by a given current upon the needle, or by decreasing the restoring force exerted by the earth's magnetic field. The first is accomplished by making the coil of a large number of turns of fine wire very much smaller in radius, and therefore much nearer the needle. The restoring force exerted by the earth may be decreased by making the needle of two small magnets, one above and one within the coil, rigidly attached together and with their poles in opposite directions.

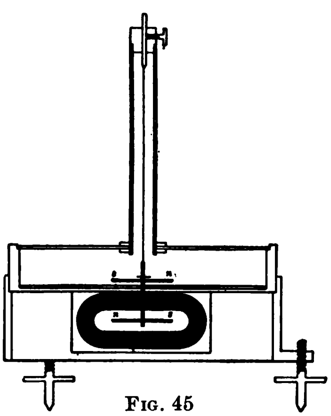

Fig. 45

A moment's consideration of this arrangement will show that the upper needle — the one above the coil — is acted upon by the current with a force tending to turn the whole system in the same direction as does the force exerted by the current upon the needle within the coil. These two forces are therefore additive. On the other hand, the force exerted by the earth on the two needles which form the system are in opposite directions. If the two

needles were of equal strength, the whole system would obviously stand with equal readiness in any position. As a matter of fact, the two needles are not exactly equal, and the directive force exerted by the earth is on the difference between the two. A second method of astaticizing is by neutralizing the strength of the earth's field by a large outside controlling magnet.

If the above adjustments are carried out with great care, the instrument becomes exceedingly sensitive and of value in detecting extremely small currents. If it is to be used for measuring currents, it must be standardized by comparison with a tangent galvanometer or its equivalent. Galvanometers in which the restoring force on the needle has been greatly diminished are said to be astaticized.

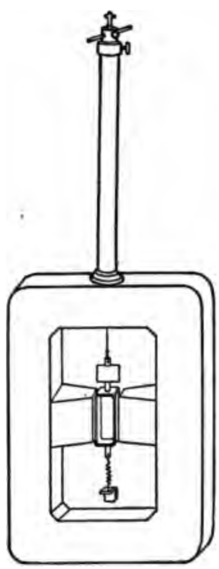

FIG. 46

D'Arsonval Galvanometer

In the D'Arsonval galvanometer, which is rapidly coming into use for the measurement of very small currents, the magnet is fixed and the coil is suspended. The suspension is made by means of very fine wire or ribbon of metal which also serves to bring the current into the coil, the current leaving the coil by a spiral wire or ribbon below. The current is detected and measured by the rotation of the suspended coil, the amount of rotation being measured by a mirror, telescope, and scale. The sensitiveness of the instrument depends upon the strength of the field between the poles of the magnet, upon the number of turns in the coil, and upon the delicacy of the suspension. If it is to be used not merely to detect currents but to measure them, it must be calibrated by comparison with some standard instrument.

AMMETERS

The commercial and more portable instruments for measuring currents are called ammeters. Of these there are a number of forms, involving entirely different principles. The most common, however, is not unlike the D'Arsonval galvanometer in fundamental principle. It consists of a coil between the poles of a powerful, permanent magnet. In order that the instrument may be portable, the coil is not suspended, but turns on jewel bearings, the current being led into and out from the coil by spiral springs. These instruments come from the maker calibrated and with the scale so graduated as to read the strength of the current directly in amperes. When the instruments are constructed for the measurement of very small currents they are generally graduated to read directly in thousandths of amperes, and are called milliammeters. The readings of the instruments, in amperes or milliamperes may obviously be reduced to absolute units by dividing by ten or ten thousand.

20. ELECTROLYSIS

When an electric current flows through an electrolyte — a compound liquid conductor — the compound is decomposed, part being liberated at the entering or positive electrode, and the other part at the negative electrode. If the liquid is a solution of silver nitrate, .001118 grams of silver are deposited per second per ampere on one of the electrodes, and the equivalent amount of acid eats away an equal amount of silver from the other. The weight of copper deposited in a given time is thus a measure of the current which has passed. This, combined with the tangent galvanometer method of measuring the

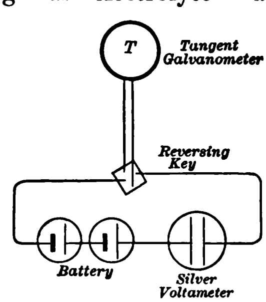

FIG. 47

current, gives an electro-chemical method of determining H. For if a steady current of electricity has been flowing for T seconds through both the tangent galvanometer and the electrolytic cell, in the former producing a deflection a of the needle, in the latter depositing w grams of silver, the following two formulæ hold:

$$C = 10 \frac{Hr}{2\,\pi n} \tan a,$$

and $\quad w = .001118\, TC, \quad$ or $\quad C = \dfrac{w}{.001118\, T}.$

Since the current is the same in all parts of the circuit,

$$10 \frac{Hr}{2\,\pi n} \tan a = \frac{w}{.001118\, T};$$

whence

$$H = \frac{w\, 2\,\pi n}{.001118\, Tr\, 10 \tan a}.$$

From this H may be computed.

The electrolytic cell in this apparatus consists of a ten per cent solution of silver nitrate contained in a platinum cup or crucible, and in which dips an electrode of chemically pure silver. The current must of course be passed through the cell in such a direction as to deposit the silver on the platinum, which is the receiving electrode. If the current is too strong the deposit of silver will not be firmly adherent to the platinum. In a preliminary experiment adjust the direction and the strength of the current, at the same time adjusting the number of turns in the tangent galvanometer so that the deflection will be in the neighborhood of 45°. This having been done and all connections made, remove the platinum cup and clean thoroughly with nitric acid, rinsing with distilled water. Dry and very carefully weigh the cup. The cup being refilled and replaced the apparatus is ready. The experiment, so far as the silver voltameter is concerned is as follows: Note the exact time at which the circuit is completed. Allow the current to flow at least half an hour. Read the galvanometer every two minutes while the current is flowing. At regular intervals reverse

the current through the galvanometer that deflections may be obtained both to the right and to the left. In every case read both ends of the needle. The average of all the readings is the angle a in the above formula. Note the exact time of stopping and the time T in seconds that the current has been flowing. Remove the receiving electrode and wash it carefully in running water without rubbing, dry, and reweigh. The gain in weight is w. Calculate H by the above formula. This is the value of the horizontal component of the earth's magnetic force at the center of the tangent galvanometer.

The experiment described above was for the silver voltameter only. In the experiment to be performed, however, it is proposed to use both a silver and a gas voltameter, the two in series with each other, to measure the strength of the current in each, the one as a check on the other, and from each to make a determination of the value of H. The details of the gas voltameter and of its use are as follows: The instrument, as shown in the adjacent diagram, consists of two vertical tubes closed with stopcocks above and joined below. Each contains a platinum electrode. It is filled by the standpipe S and emptied when necessary by the stopcock at the bottom of the U-tube. The apparatus is to be filled with dilute sulphuric acid of ten per cent strength, chemically pure sulphuric acid and distilled water being used. The gases generated by the current are collected and measured in the vertical tubes. At the beginning of the experiment the dilute sulphuric acid should entirely fill the two tubes. At the close of the experiment the acid is allowed to flow out of the stopcock until its level in the hydrogen tube is the same as that in the standpipe. The barometric pressure on the gas is then the same as that of the outside atmosphere. The barometer and the temperature having been read, the volume of the contained

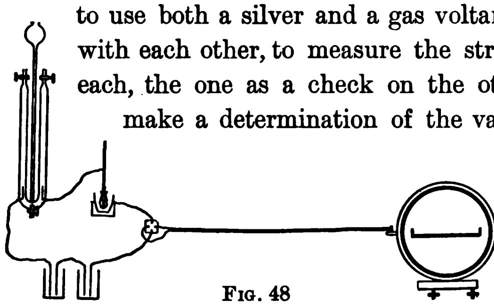

FIG. 48

gas under normal conditions, that is to say, at a pressure of 76° and a temperature of 0°C., can be calculated by the familiar method. There is one, but only one correction which has not been thus taken into consideration. The gas is not all hydrogen, but a mixture of hydrogen and vapor of water. The part of the barometric pressure on the mixture supported by the vapor of water depends upon the temperature and the strength of the acid solution. Table 3 in Appendix III gives the barometric pressure sustained by the vapor of water over ten per cent acid solution at different temperatures. If the number determined from this table is subtracted from the barometric pressure, the result will be the barometric pressure on the hydrogen alone and the pressure which should be used in determining the volume which the collected hydrogen would have at standard conditions as to pressure and temperature. The amount of hydrogen liberated per ampere per second under standard conditions as to temperature and pressure is .1156 cc.

Join in series the silver voltameter, the gas voltameter, a resistance box of a low grade, two or three storage cells (as the experiment may require), and a tangent galvanometer. A low grade resistance box is used because of the danger of burning out and thus ruining a fine one. The tangent galvanometer should be reached through a reversing key, and the current should be led to the tangent galvanometer (here as always at a considerable distance from the rest of the apparatus) through twisted wires.

The details of the experiment have already been described in the discussion of the silver voltameter. Determine the value of H, using both the silver and the gas voltameter. The two results should of course agree. Note the table and the position on the table at which the galvanometer stands. This constitutes the record of the point in the room at which the value of H has been determined.

21. EQUIPOTENTIAL LINES AND LINES OF FLOW

When a current of electricity enters a broad but thin conducting surface it spreads throughout every part of it, forming what is called a current sheet. Imaginary lines from one electrode to the other, following the direction of the current, are called lines of flow. Two points on such a sheet are said to be at the same potential, if no current will flow through an outside metallic circuit from one to the other. Lines connecting points at the same potential are called equipotential lines. Lines of flow and equipotential lines cross everywhere at right angles.

Connect the poles of a Daniell cell with two points upon opposite ends of a thin plate of carbon. Touch the sheet at one edge with one terminal *a* of a sensitive galvanometer *G*, move the other terminal *b* up and down on the line two centimeters over. Find a point such that when it is touched by the terminal *b* the galvanometer shows no deflection. With *a* remaining in the same position, find a similar point on the line four centimeters over, and so on across the sheet. These points are at the same potential. Locate the points on coördinate paper ruled into similar squares, and draw a curve connecting them, thus forming an equipotential line. Move the terminal *a* two centimeters along the edge and repeat. In this way find equipotential lines over the whole surface and plot them on coördinate paper.

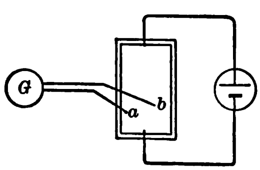

FIG. 49

Draw on the coördinate paper from one pole to the other several lines of flow, the rule being that they cross the equipotential lines everywhere at right angles.

An interesting variation of this experiment can then be made by an irregularly shaped piece of carbon or one ground away to excessive thinness over more or less extended regions. The result will be a peculiar and interesting distortion of the equipotential lines and lines of flow.

22. WHEATSTONE'S BRIDGE

Wire or British Association Form

If part of a circuit carrying a current is divided, there is a continuous fall of potential on both branches from the point at which they separate to the point at which they reunite. For every point on one branch a corresponding point on the other may be found having the same potential. If two such points (m and n) be connected by a wire no current will flow, and if in this circuit a galvanometer G be inserted, its needle will not be deflected. Denote the resistance of the various parts by the letters a, b, x, and r. By Ohm's law the current in any part is equal to the difference of potential divided by the resistance, and as no current is carried off by the bridge, the current in b is equal to that in a; hence

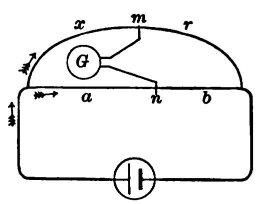

Fig. 50

$$\frac{\text{the fall of potential in } a}{a} = \frac{\text{the fall of potential in } b}{b},$$

or

$$\frac{\text{the fall of potential in } a}{\text{the fall of potential in } b} = \frac{a}{b}.$$

Similarly in the other branch, the current in r is the same as in x, and hence

$$\frac{\text{the fall of potential in } x}{\text{the fall of potential in } r} = \frac{x}{r}.$$

Since m and n are at the same potential, the fall in a is equal to the fall in x, and the fall in b equals the fall in r. Hence the first members of the last two equations are equal numerator to numerator and denominator to denominator. The second members are therefore equal,

$$\frac{x}{r} = \frac{a}{b} \quad \text{or} \quad x = \frac{a}{b} r.$$

If the resistance r and the ratio $\frac{a}{b}$ are known x can be calculated.

For the branches *a* and *b* a simple straight wire can be used, and the ratio of the lengths of *a* and *b* may be taken for the ratio of their resistances, since the two are proportional. For *r* use ohm coils, the ohm being the practical unit of resistance.

Connect a wire bridge, as shown in the adjacent diagram, and measure the resistance of a number of coils, comparing them with various standard coils, the adjustment being made by moving the sliding contact until the galvanometer shows no deflection when its circuit is closed. A double key should be so inserted that on being pressed it will close first the battery and then the galvanometer circuit. This will prevent unnecessary waste of the battery. To correct in part for contact resistance at the ends of the meter wire, interchange the known and unknown coils and repeat each measurement. Take the mean of the two measurements. Interchange the galvanometer and battery, and repeat, thus illustrating what is called the conjugate property of the bridge.

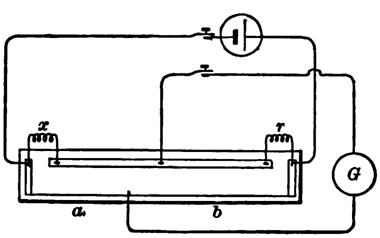

FIG. 51

The more nearly *r* is taken equal to *x*, the nearer the bridge will be to the center of the wire, and the more accurate will be the result. The wire bridge is especially adapted to the measurement of low resistances. The galvanometer should also be of low resistance; if formed of several coils, these should be joined in parallel. The coil whose resistance is to be measured should be so placed that the current flowing through it will not affect the galvanometer directly. This may be tested by disconnecting the galvanometer and making and breaking the current through the rest of the apparatus. In all electrical work the student should bear in mind that good contact can only be secured when the pieces of metal are bright. The wires should therefore be scraped before a joint is made, and the binding posts should be occasionally cleaned.

Box Form of Bridge

For measuring high resistances the box bridge with a high resistance galvanometer is more desirable than a wire bridge, for in it all the known resistances can be made to approach more nearly the magnitude of the unknown resistance. Here the ratio a to b is definitely fixed by known resistances to any desired ratio, 1 to 10, 1 to 100, or 1 to 1000 at pleasure, and the resistance r is then varied until balance is obtained. The formula is the same as with the wire bridge, $x = r\frac{a}{b}\cdot$ The galvanometer G should be of many turns of wire and therefore of high resistance. The conjugate property of the bridge, the interchangeability of galvanometer and battery connections, was illustrated in the last experiment. It can be proven mathematically or shown experimentally that the best arrangement in this case is that in which the galvanometer connections reach from the junction of the highest two resistances to the junction of the lowest two. The second arrangement, in which the galvanometer and battery have been interchanged, is therefore used.

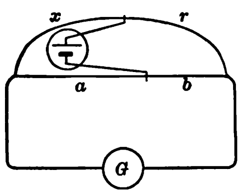

Fig. 52

Box bridges vary greatly in the grouping of their resistance coils; but by first drawing a diagram of the ideal bridge, as above, and also of the resistance box, we may proceed as follows to complete the connections. Choose for a and b that portion of the box the coils of which progress decimally, 1, 10, 100, etc., and for r usually the remainder of the box having the greatest range and capable of the most gradual variation. Consult before each step the ideal diagram, Fig. 52; a and b, if not already connected, should be joined by a plug if possible, if not, by a

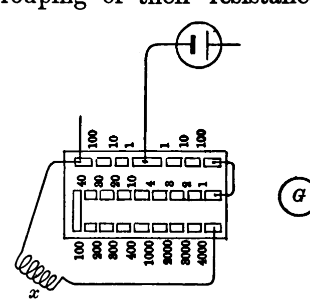

Fig. 53

short, heavy strip of copper; join one end of r to b; to the other end of r join x, and the other end of x to a; where a and b join attach one terminal of the battery, and proceeding in this way complete the connections.

Measure carefully the resistance of a coil, starting with a ratio $\frac{a}{b}$ of 1 : 1, then of 1 : 10, and, if possible, of 1 : 100. It will not generally be possible to secure exact balance, but two values of r may be found differing by one ohm, which will give deflections in the galvanometer in opposite directions. The correct value of r lies between. Finally, at the highest ratio of $\frac{a}{b}$ find the values of these deflections in opposite directions, and interpolate for the value of r, which would not produce a deflection in either direction.

The taper plugs in the resistance box must be turned in firmly to avoid what is called plug resistance. At the end of the experiment, however, the plugs should be loosened in order to relieve the strain on the cover of the box.

23. MEASUREMENT OF RESISTANCE IN ABSOLUTE UNITS

When two points are so circumstanced that anything, whether running water, magnets, or current electricity, will do work in moving from one to the other they are said to be at different potentials. When a current of electricity flows through a wire work is being done, appearing ordinarily in the form of heat. Therefore, two points along the wire are said to be at different potentials. They are at the absolute unit difference of potential when the absolute unit of current will do the absolute unit of work per second in flowing from one to the other; and the wire between the two points has the absolute unit of resistance. If C units of current are flowing between two points whose difference of potential is E, the work done per second is $W = CE$.

Pass the current from a storage battery through a spiral of fine German silver wire, wholly immersed in water. Include in the circuit, which should be of large wire, a reversing key and a tangent galvanometer. Allow the current to flow, noting at regular intervals the deflection of the galvanometer, until the temperature of the water has risen from t_1 to t_2, about 10°. Record the time — T seconds — required, and the weight w of water. Calling w_1 the thermal capacity of the inner vessel, the quantity of heat generated is $(w+w_1)(t_2-t_1)$. From the experiments of Joule it is known that 41,600,000 absolute units of work, called ergs, are required to produce one unit of heat. Therefore, the heat given to the water must have been generated by

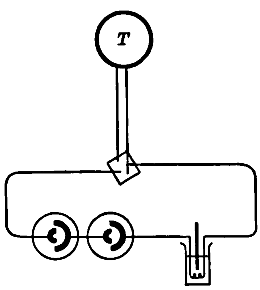

FIG. 54

$$W = 41{,}600{,}000\ (w + w_1)\ (t_2 - t_1)$$

ergs of work done in the wire. But the work done in the wire per second is CE, and during T seconds

$$W = CET.$$

And since by Ohm's law $E = CR$,

substituting, $$W = C^2RT.$$

Equating $$C^2RT = 41{,}600{,}000\,(w + w_1)\,(t_2 - t_1),$$

or $R = \dfrac{41{,}600{,}000\,(w + w_1)\,(t_2 - t_1)}{C_2T}$ absolute units of resistance.

Calculate C in absolute units from the tangent galvanometer deflection (see p. 65), and substituting in the above equation find R. In this way measure the resistance of the coil.

Measure the resistance in ohms by the wire bridge and compare the results, and thus obtain the value of an ohm in absolute units of resistance.

In the equation $C = \frac{E}{R}$ substitute for C the number of absolute units of current in one ampere, for R the number of absolute units of resistance in an ohm, and the result will be the number of absolute units of electromotive force in a volt.

24. ELECTROMOTIVE FORCE OF A DANIELL CELL BY THE POTENTIOMETER

The advantage of the following method of determining the electromotive force of a cell is that the current flowing through the cell is zero, and that the cell is therefore free from polarization. The adjacent diagram shows the arrangement of the apparatus. C is a Daniell cell, whose electromotive force is being determined; B is a battery, maintaining a constant current flowing through a wire. The electromotive force of the battery B must be greater than that of the Daniell cell. There

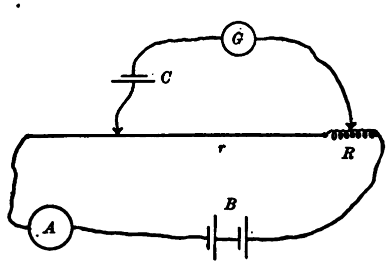

FIG. 55

is, then, through the wire a total difference of potential greater than the difference of potential of the terminals of the cell C. If the terminals of the cell C are connected to two points on the wire such that the difference of potential between the two points is equal to the difference of potential of the terminals of the cell, no current will flow through the leads. The absence of this current will be indicated by the galvanometer G. The strength of the current flowing through the wire is determined by means of a galvanometer, or ammeter in series with it, indicated in the diagram by the letter A. If the resistance of the wire per centimeter of length is known, the total resistance between the two points of contact is directly calculable. The product of the current in amperes

and the resistance in ohms will give the difference of potential between the two points of contact in volts. Measure the resistance of the wire by an auxiliary Wheatstone's bridge, and then perform the above experiment. Determine the electromotive force of the Daniell cell.

Another method of determining the electromotive force of the cell, which has the disadvantage that a current is flowing through the cell, is as follows:

Join in series, as shown in the adjacent diagram, the Daniell cell, an ammeter, and a resistance box. Vary the resistance in the box from one to four ohms by steps of one ohm, measuring the current corresponding to each resistance, indicating corresponding currents and resistances by r_1, r_2, r_3, etc., and c_1, c_2, c_3, etc. The following formulæ, which merely express Ohm's law, hold:

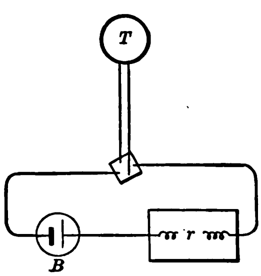

FIG. 56

$$c_1 = \frac{E}{r_1 + y}, \qquad c_2 = \frac{E}{r_2 + y}, \text{ etc.}$$

In these equations y is the resistance of the circuit outside the resistance box. It is unknown. Any two of these equations will, by the diminution of y, determine the value of E. Pairing the equations in various ways, calculate the value of E, and find the average of the several results.

This method, which is known as Ohm's, will serve as a check on the electromotive force as determined in the preceding experiment. The method, however, is not in every way satisfactory, the principal difficulty being that there is, during the whole experiment, a current flowing through the cell, and that the cell is therefore more or less polarized. As the current changes, the amount of polarization is changed, and as the method assumes that the electromotive force remains constant, the result is, to a certain extent, at fault.

25. THERMO-ELECTROMOTIVE FORCE

When two different metals are in contact there is at the point of contact a difference of potential. The difference of potential is a function of the temperature. If two strips or wires of different metals are joined together at both ends, and if the two ends are kept at different temperatures, there thus results an electromotive force, called thermo-electromotive force, and therefore a current in the closed circuit. Such a pair of metals is called a thermal couple. The thermo-electromotive force, and therefore the current, is equal to the difference of temperature multiplied by a number which is a constant so long as the mean between the temperatures of the hot and cold junctions is constant.

For the most direct investigation of this phenomenon one of the metals may be copper, and the galvanometer, which is wound with copper wire, may be included in this part of the circuit. The metal to be paired with copper, for example German silver, *G. S.*, may be made to connect the two terminals, as shown in the diagram. The junctions should be soldered together and protected from the water in which they are heated or cooled by being inclosed in glass tubes. When thus immersed the wires should touch only at the bottom, where they have the temperature of the baths. Above they should be separated by cotton packing, which, filling the tops of the tubes, will prevent draughts of air down the tube. If the resistance of the galvanometer is sufficiently great, the resistance of the short metal joining the two junctions may be neglected. The deflections of the galvanometer are then proportional to the electromotive force developed, and the galvanometer may be calibrated by a potentiometer method shown in Fig. 58. The resistance R is large, several hundred ohms, while r is much smaller, part of a

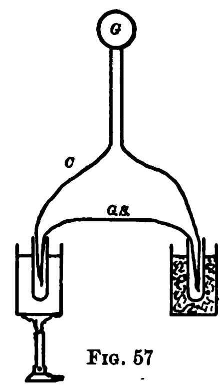

FIG. 57

stretched wire. D is a Daniell cell, whose electromotive force E is known from a previous experiment. The difference of potential at the terminals bears to E the ratio of r to the resistance of the whole circuit $R + r + B$, — practically R if R be very large. The galvanometer, G, having been calibrated, the electromotive force developed by the couples can be calculated. This should be expressed in volts per degree difference of temperature of the terminals.

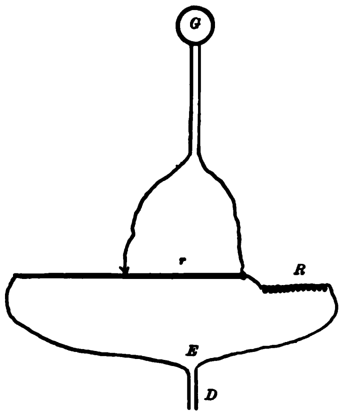

FIG. 58

Determine the electromotive force per degree difference of temperature of a copper German silver, a copper iron, and a copper lead couple over a range from 0° to 100° C.

26. VACUUM-TUBE PHENOMENA

When an electric discharge takes place in a gas, for example air, the character of the phenomenon depends upon the pressure of the gas and also, to a certain extent, upon the shape of the tube in which the discharge takes place.

A mercury air pump best serves to carry the exhaustion of the air to very low pressures. This has so many forms that no attempt will be made to give a general description or discussion of it. The method of manipulating any particular pump and the principle on which it acts can generally be discovered by an examination of the instrument itself.

In order to measure the pressure of the air, we shall have occasion to use in this experiment three types of gauges, according as the pressure is only slightly reduced, moderately reduced, or reduced to a very low value indeed. The first consists in a tube dipped in

a vessel of mercury, the upper end of the tube being connected to the reservoir whose pressure is to be measured. The difference between the height of the mercury in this tube and the barometer reading at the time gives the pressure. This is useful for all pressures from 6 or 7 cm. up to full atmospheric pressure. Second, below 6 or 7 cm. and down to a millimeter or so the pressure may be measured by a device whose action is pretty well explained by the left-hand figure in the adjacent diagram. The tube at *A* is connected with the air pump, the lower part of the tube and the closed side being filled with mercury. When the pressure is less than the difference between the height of the closed end and the level of the mercury in the other arm of the U-tube the mercury will sink in the closed arm. The difference in level of the mercury in the two arms of the tube will then give the pressure of the air in the pump. Third, when the pressure is very low indeed neither of the above devices is sufficiently delicate, and it is necessary to have recourse to a McLeod gauge, which is shown in the right-hand figure. The gauge is connected at *A* to the air pump or reservoir whose pressure is to be made. *M* leads to a mercury reservoir, which being raised or lowered will force mercury into the gauge. In order to measure any pressure, the mercury is lowered until the large bulb *M* is in free connection with the air pump. This connection must be left open for a little while, in order that the pressure may equalize throughout the pressure; the lower the pressure, the longer this takes, as diffusion gets less and less rapid. The mercury is then raised, cutting off the bulb *M* and entrapping in it a certain quantity of air, which as the mercury continues to rise is compressed

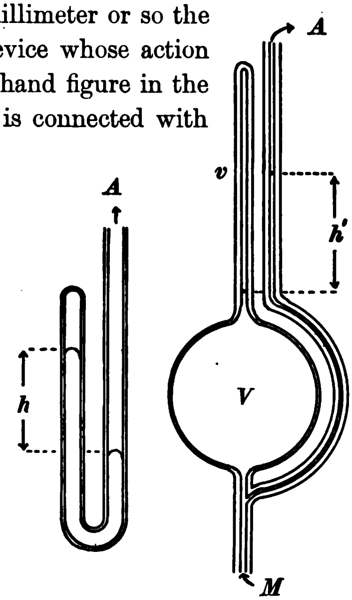

FIG. 59

into smaller and smaller space. Since the pressure of the gas is in inverse proportion to its volume, the difference in level of the mercury in the tube leading to the air pump and in the bulb M increases as the volume of the air gets less. The mercury is raised in the bulb M until it reaches a small fine mark on the extension tube. Before the gauge was put on the air pump, the volume v of the extension tube above the mark and the volume V of the bulb above the point where the tube separates to go to the air pump must have been determined. These being known and also the difference in the level of the mercury in the bulb and in the side tube leading to the air pump, the initial pressure of the air in the bulb and thus in the pump is readily calculable.

Having an air pump provided with these three methods of measuring pressure, connected hermetically with a vacuum discharge tube, vary the pressure in the discharge tube and observe the succession of phenomena on passing through it an electric discharge obtained from an induction coil. Measure and record the pressure corresponding to each change in appearance, or the limits in pressure between which each phenomenon persists.

27. EARTH INDUCTOR AND DETERMINATION OF DIP

Whenever a conductor is moved in a magnetic field in such a way as to cut across the lines of magnetic induction a difference of potential is established in the conductor which is proportional to the rate at which it is cutting the lines of magnetic induction. In estimating the rate of cutting when the process is complicated, either because of peculiarity in the shape of the conductor or in the magnetic field, the reckoning is to be algebraic, that is to say, a reversal of direction of the lines of force or of the direction of cutting reverses the potential. If the conductor forms a closed circuit of constant resistance, a current flows which is proportional to the difference potential induced, unless, indeed, the variation is so rapid that the self-induction of the circuit exerts a perceptible effect.

If a coil of wire is rotated in the earth's magnetic field, a current is induced in the coil which reverses in direction as the coil passes through the position in which it is perpendicular to the plane of the earth's magnetic meridian. During each half of the revolution the quantity of electricity which flows through the circuit is proportional to the area of the coil, to the number of turns in the coil, and to the component of the earth's magnetic force perpendicular to the axis of rotation. A coil thus used is called an earth inductor. It is usually so mounted that it can be turned only through one half a revolution. The quantity of electricity induced by this half turn of the coil is measured by a ballistic galvanometer. Thus connected the coil can be used to determine the dip of the earth's magnetic field. For this purpose the earth inductor must be so mounted that it is possible not only to turn the coil through a half revolution about an axis in the earth's meridian, but also so that the position of this axis can be adjusted to any angle in the meridian. The adjacent diagram shows an earth inductor so mounted as to have two axes of rotation for the coil. One axis is horizontal, and is provided with a divided circle for setting the coil and a clamp for holding the position; the other axis is at right angles and permits of a half revolution accurately determined by stops.

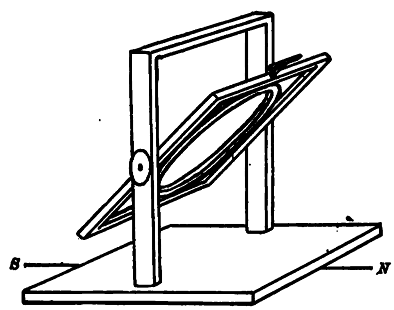

Fig. 60

Using a small compass, set the horizontal axis at right angles to the earth's magnetic meridian. Clamp the horizontal axis so that the perpendicular axis will be vertical. Turn the coil quickly but not violently through 180° about the vertical axis and note the throw in the ballistic galvanometer. This will be proportional to the horizontal component of the earth's magnetic force H. Turn the coil through an angle of 10°, as shown by the divided

circle on the horizontal axis. The coil being back in its original direction (it is necessary for accuracy that the deflections of the ballistic galvanometer should all be in the same direction) and the galvanometer being at rest, turn the coil again through 180° and note the throw in the galvanometer. Repeat at intervals of 10° through a complete revolution. Plot the results on coördinate paper, angles being plotted as abscissæ, and the throws of the ballistic galvanometer as ordinates. This curve is interesting as being a sine curve, and, in its particular application in electricity, as being at the foundation of the theory of alternating current machinery.

From this curve deduce the angle of dip of the earth's magnetic field.

Repeating the above work for the horizontal and vertical component of the earth's magnetic field, taking a number of throws of the galvanometer for each in order to secure greater accuracy, calculate from the relative magnitudes of these two components the direction of the earth's force, — in other words, the dip.

Compare the two determinations of the dip.

The earth inductor is used to determine the magnetic induction in various parts of commercial instruments, as, for example, in the parts of a dynamo, in the space between the field coils and the armatures, or back of the field coils, as well as leakage at various points. For this purpose the earth inductor and an exploring coil are joined in series, and in series also with the ballistic galvanometer. The earth inductor fixed in a vertical position, and therefore in its revolution cutting only the horizontal component of the earth's magnetic force (H, taken as known), gives a standard with which to compare the induction through the small exploring coil. In a half turn of the earth inductor the total magnetic induction cut is $2HNA$,

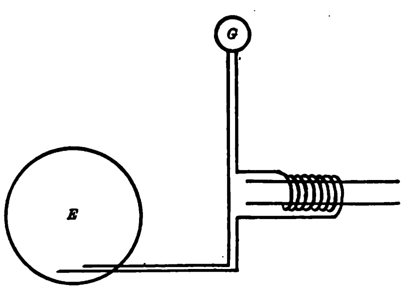

FIG. 61

when N is the number of turns in the coil and A the mean area of the coil. The change in magnetic induction through the exploring coil can therefore be calculated by comparison, the two being approximately proportional to the throws. If the exploring coil is rotated 180° in the field which is being explored, each turn of the coil cuts the induction through it twice, so that the strength of the field is that calculated from the ratios of the throws in the ballistic galvanometer divided by twice the number of turns. The same is true if by reversing the current of the field coils the direction of induction (or flux) through the coil is reversed. If, however, the coil is merely quickly removed from the field, the induction through it is cut only once by each turn.

Following the lines of procedure thus suggested, measure the magnetic induction through the armature of a small dynamo and also through the yoke which connects the field coils together at the back. For this purpose it is necessary to wind one's own exploring coils.

28. THE DYNAMO

The machine used in this experiment is ordinarily called a motor generator. One half of the machine is an alternating current motor and is here used only as a source of power. It is directly connected to the dynamo, the study of which forms the essential part of the present experiment.

Draw a diagram showing all the parts and connections of the dynamo. This diagram should show clearly the direction of winding, the polarity of the iron, and the direction of the current, as indicated by arrows, flowing through the several parts. The polarity of the iron can be determined by means of a small pocket compass. The direction of the current in parts of the circuit can be determined in the same way by holding the compass immediately under the wire carrying the current. The direction of deflection of the north-seeking pole will indicate, by an application of the ordinary rule, the direction in which the current is flowing.

The direction of the current in one or two parts of the circuit having been determined, it is of course possible to trace back what must be the direction of the current in the other parts.

Connect the dynamo so that the current from it will flow through the ammeter and through some incandescent lamps submerged in a vessel of water. Connect the voltmeter so that it will measure the voltage at the terminals of the lamp. The rise in temperature of the water enables one to calculate, provided he determines the weight of water and the thermal capacity of the calorimeter, the amount of heat generated in the lamps, and thus to calculate the work done in lighting the lamps (see Experiment 23). In order to make this experiment accurate, it is of course necessary to determine the amount of heat radiated in a manner similar to that in the experiment on the mechanical equivalent of heat. Allow the current to flow until the rise in temperature has been 10°, taking care to keep the water thoroughly stirred; then, having stopped the current, allow the water to cool, determining the rate of radiation; and thus the average rate of radiation. From this the total rise in temperature which the water would have undergone, had there been no loss by radiation, can be determined. Calculate from this the work done in lighting a single lamp for one hour, expressing the result in horse power. One horse power is equal to 7.46×10^9 ergs per second.

Remove the ammeter and voltmeter from the external circuit. By means of the two instruments thus freed, measure the current and voltage in the different parts of the circuit, — the field coils, the armature, and the field resistance. To do this, place the ammeter directly in the circuit and place the terminals of the voltmeter at the ends of that portion of the circuit in which it is desired to know the voltage. From a knowledge of the strength of the current and the voltage, calculate the resistance in ohms of the several parts of the circuit $C = \frac{E}{R}\cdot$ In the field rheostat measure not only the resistance of the whole rheostat but the resistance of each section in it.

APPENDIX

APPENDIX

I. SIGNIFICANT FIGURES

An investigator who, in publishing the results of his experiments, carelessly keeps a greater (or a less) number of figures than are known not only deceives himself, but often misleads others in regard to the accuracy to which he has attained. There are other methods of discussing results, but the following is sufficient for the purposes of the present course. It may perhaps be best illustrated by taking the first experiment as an example.

The measurement of the dimensions of a cylinder by a vernier gauge gives as its length 5.215 cm., and as its diameter 3.020 cm. The vernier gauge cannot measure accurately thousandths of a centimeter, and the last figure in each of the above values is in great doubt. The true length and the true diameter of the cylinder may be either greater or less than these values by .002 cm. To calculate the volume of the cylinder we have the formula $V = l\pi r^2$. The radius is 1.510. The arithmetical work is given in full, as it is most instructive.

$$
\begin{array}{r}
1.51\mathbf{0} \\
\underline{1.51\mathbf{0}} \\
\mathbf{0000} \\
151\mathbf{0} \\
755\mathbf{0} \\
\underline{151\mathbf{0}} \\
2.28\mathbf{0100}
\end{array}
$$

Since the last figure in the value of the radius, the cipher, is in doubt, all of the figures printed in heavy-faced type are in doubt. The first cipher in the product is in doubt, therefore the succeeding figures are wholly unknown, and should be discarded. Proceeding in the calculation:

```
     2.280
     3.141
    ------
      2280
     9120
    2280
   6840
   --------
   7.161480
```

Again discarding the figures which are wholly unknown, and keeping only the first figure which is in doubt,

```
      7.161
      5.215
     ------
      35805
      7161
    14322
   35805
   ---------
   37.344615
```

The final result is to be entered as 37.34, the succeeding figures, which are wholly unknown, being discarded. That there has been a great saving of labor will be obvious, if the student will but repeat the above calculation without discarding any of the figures.

If the calculation is by logarithms, this method of discussing the results is not practicable, and the following method may be employed. If the calculation involves only multiplying and dividing, the percentage error introduced into the result by any factor is equal to the percentage error in that factor. Thus, in the above example, the error which could pass undetected in the measurement of the length is supposed to be .002 cm.; this is .04 per cent of 5.215. Therefore, the possible error in the result arising from this source is .04 per cent of 37.3446 ··· or .014 cc. Obviously, therefore, the first figure 4 in the result is slightly in doubt, and the subsequent figures are unknown and must be discarded. If one of the factors is squared the percentage error in the result is twice as great. If a factor is cubed its error is tripled in the result. If the square root of a factor is taken the possible error is halved in the result. Thus, in the above calculation of the volume of the cylinder the radius is squared. The error in the measurement of the diameter which could pass undetected is assumed to be .002 cm. This is nearly .07 per

cent of 3.020, — the diameter. Since the radius is squared, the possible percentage error arising in the result from this source is .14 per cent; .14 per cent of 37.3446 cc. is .05 cc. Thus the first figure 4 in the result is in doubt, subsequent figures are unknown, and the result should be written 37.34.

If several equally careful observations have been made of the same quantity, the mean of them all is what Holman calls the "best representative measure." The error of this mean depends upon the errors in the individual observations. These errors are of two kinds, — constant errors that affect all observations equally and in the same direction, and errors of such a nature that they are variable and as likely to be positive as negative in sign. The mean of many observations is of course no better than a single observation as far as errors of the first type is concerned, but in respect to errors of the second kind, the mean is probably more accurate than the single observation and its "probable error" may be calculated in the following way. Find the difference between each observation and the mean, this being called the deviation of the single observation. Find the average deviation which is denoted by a.d. Let n be the number of observations. Then the probable error of the mean of all the observations P.E. is given by the formula $\text{P.E.} = .84 \frac{\text{a.d.}}{\sqrt{n}}$. A very clear and practical discussion of this subject may be found in Holman's *Discussion of the Precision of Measurements.*

II. GRAPHICAL REPRESENTATION

When a series of results depends upon some one regularly varying factor in the experiment a very instructive graphical representation may be made by plotting the results on coördinate paper. This method of recording results may be most simply illustrated by an experiment in which the temperature of a cooling body has been observed at frequent intervals: in this the temperature observations form the series of results, depending upon the time which has elapsed as the one varying condition. In the following the first column of figures gives the time in minutes which had elapsed from the beginning of the experiment, and the second column the corresponding temperature observations.

TIME	TEMPERATURE
2 min.	73° C.
6	65°
8	58°
13	51°
18	45°
24	38°

On the coördinate paper, which is ruled by numerous equally spaced lines, a certain horizontal line is chosen as the "axis of abscissæ" and a certain vertical line as the "axis of ordinates"; the intersection of the two lines is called the origin. A horizontal distance measured along the axis of abscissæ is to represent the time from the beginning of the experiment to a certain observation, and a vertical distance measured from the point thus reached is to represent the corresponding temperature observations, each according to some chosen scale. Each observation is to be plotted in this way and a curve drawn through the resulting points. In the curve plotted below, each

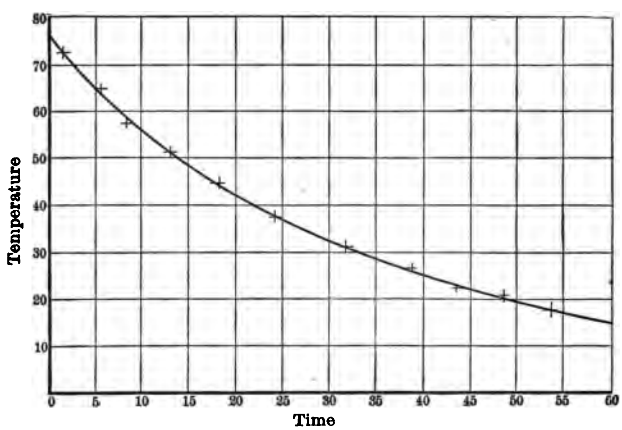

FIG. 62

horizontal space represents 5 minutes, and each vertical space represents a change of temperature of 10°.

It is not always desirable, or possible, to draw the curve through all of the points marked, but if it is known that the curve should be

a smooth curve, and if the points are numerous, it should be drawn between them, thus averaging, in a measure, the errors of observation.

If the quantity plotted is large in comparison with its variations, the scale on which the curve is plotted must be correspondingly large in order that it may show these variations. This may be done without using an awkwardly large sheet of paper by plotting only the curve and dispensing with the paper below. This is illustrated in the following curve, representing the load required to sink a Nicholson hydrometer in water at various temperatures.

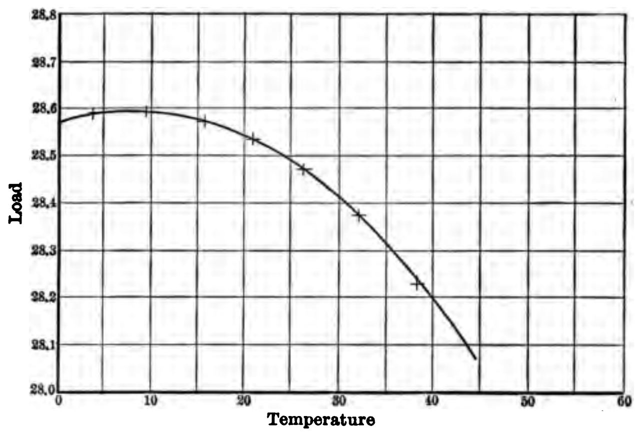

Fig. 63

The scale on which the curve is drawn should always be indicated along the two axes.

If a quantity is being plotted as ordinates, some of whose values are positive and some negative, the positive values are plotted upward from the axis of abscissæ, the negative values downward. Thus, in the first example, if the body had cooled below zero, the curve would have crossed the axis of abscissæ. When the abscissæ are negative they are to be plotted off to the left from the origin.

Curves are of especial service as quick and accurate means of interpolation. Thus, from the curve, the temperature of the cooling body fifteen minutes after the beginning of the experiment was 48° C.; the weight required to sink the hydrometer to the given mark on the stem in water at the temperature of 19° C. would be 28.55 grams.

III. TABLES[1]

1. *Weight in grams of one cubic centimeter of dry air at different temperatures and barometric pressures*

Note. Two ciphers are to be prefixed to all the numbers in the body of the table.

Temperature (Centigrade)	Barometric Pressure[2] (centimeters of mercury)							
	70 cm.	71 cm.	72 cm.	73 cm.	74 cm.	75 cm.	76 cm.	77 cm.
10°	.001149	1165	1181	1198	1214	1231	1247	1263
11°	1145	1161	1177	1194	1210	1226	1243	1259
12°	1141	1157	1173	1189	1206	1222	1238	1255
13°	1137	1153	1169	1185	1202	1218	1234	1250
14°	1133	1149	1165	1181	1197	1214	1230	1246
15°	1129	1145	1161	1177	1193	1209	1225	1242
16°	1125	1141	1157	1173	1189	1205	1221	1237
17°	1121	1137	1153	1169	1185	1201	1217	1233
18°	1117	1133	1149	1165	1181	1197	1213	1229
19°	1113	1129	1145	1161	1177	1193	1209	1224
20°	1110	1125	1141	1157	1173	1189	1204	1220
21°	1106	1121	1137	1153	1169	1185	1200	1216
22°	1102	1117	1133	1149	1165	1181	1196	1212
23°	1098	1114	1130	1145	1161	1177	1192	1208
24°	1094	1110	1126	1141	1157	1173	1188	1204
25°	1091	1107	1122	1138	1153	1169	1184	1200
26°	1088	1103	1118	1134	1149	1165	1180	1196
27°	1084	1099	1114	1130	1145	1161	1176	1192
28°	1080	1095	1110	1126	1142	1157	1172	1188
29°	1077	1091	1107	1122	1138	1153	1169	1184

[1] Taken from Kohlrausch's *Physical Measurements* and Whiting's *Physical Measurements* (which contains very complete and excellent tables), and from the tables of Landolt and Börnstein.

[2] In making barometer readings it is customary to read to the top of the curvature of the mercury, and this is sufficiently accurate for the purposes of the present course, but it is well to bear in mind that when greater accuracy is to be obtained corrections must be applied for the depression by capillarity, for the temperature of the mercury and of the measuring scale, and for the force of the earth's attraction at the particular locality in which the experiment is being performed.

If the air is not dry a correction must be applied to the above values. The presence of the vapor of water as a part of the atmosphere decreases the weight per cubic centimeter. For various dew points the values of this correction are as follows.

Dew Point	Subtract
0°	.000 003
5°	.000 004
10°	.000 006
15°	.000 008
20°	.000 010
25°	.000 014

The main use of the above table is in caculating the buoyancy of the air. If the object being weighed does not occupy over 100 cc. of space, and if the weighings have not been carried further than milligrams, the above correction for the humidity of the air can usually be neglected.

2. *Space in cubic centimeters occupied by one gram of water at different temperatures*

0°	1.000 12	10°	1.000 26	20°	1.001 73
1°	1.000 07	11°	1.000 35	21°	1.001 94
2°	1.000 03	12°	1.000 46	22°	1.002 16
3°	1.000 01	13°	1.000 57	23°	1.002 38
4°	1.000 00	14°	1.000 70	24°	1.002 62
5°	1.000 01	15°	1.000 85	25°	1.002 87
6°	1.000 03	16°	1.001 00	26°	1.003 13
7°	1.000 07	17°	1.001 16	27°	1.003 39
8°	1.000 12	18°	1.001 34	28°	1.003 67
9°	1.000 18	19°	1.001 53	29°	1.003 95

3. *Vapor tension of a 10% solution of sulphuric acid and water*

Temperature	Vapor Tension in cm.	Temperature	Vapor Tension in cm.
15	1.15	20	1.58
16	1.23	21	1.67
17	1.31	22	1.77
18	1.39	23	1.88
19	1.48	24	1.99

ANNOUNCEMENTS

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